MATHEMATICS (US)

Paper 0444/23
Paper 23 (Extended)

Key messages

It is important for candidates to show each single step in a method and they should avoid trying to do more than one step on each line. Final answers should be checked to see if they are in the form required, in the correct order, and to the accuracy requested.

General comments

In longer problems candidate should keep to as much accuracy as they can in the intermediate steps, using fractions or surds where possible, and only write their final answer correct to a certain number of figures. In a single right-angled triangle, only one of the three trigonometric ratios, sine, cosine or tan, will normally need to be used to solve the triangle. The sine rule or cosine rule are more appropriate for triangles which do not have a right angle and candidates should look to improve their awareness at identifying which rule needs to be used for which scenario.

Comments on specific questions

Question 1

This question was answered very well. The two common errors were either giving the decimal 0.2 or carry out the calculation the wrong way around.

Question 2

Again, this was answered well. The common incorrect answer being 132.

Question 3

Here a few incorrect answers were seen, generally either both 13 and 15 given together or a single answer of 15.

Question 4

(a) It was not uncommon to see only one answer ringed, usually >.

(b) Very few answered this wrongly, but some candidates did not use brackets and instead changed the second subtraction sign into an addition sign to give the answer 7; 7 – 3 + 1 + 2 = 7.

Question 5

The most common errors were multiplying incorrectly to give 380.50 or to divide and give 318.18.

Question 6

This question was well answered with the most common correct method being to find 15 per cent of 400 as 60 and then to subtract it. A few did the calculation wrong and reached 80, so giving an answer of 320.

Question 7

Most candidates who achieved full marks for this question did not show any working. The common incorrect answers seen were 8^{28} and $8g^8$.

Question 8

The candidates found this question difficult. The common incorrect method was to do 8 × 4 + 9 × 3, then to divide by 17.

Question 9

This was answered well, with only a few candidates trying to work it out as a decimal.

Question 10

(a) Almost all responses were correct.

(b) Many answers were correct with little or no working shown. The common incorrect answer was $n - 4$.

Question 11

(a) Most of the candidates gave the correct answer to this question and showed their working.

(b) Most correct answers were in the form $\sqrt{\frac{P}{M} - h^2}$ seen on the answer line. Sometimes this was seen in the working and then spoilt on the answer line by writing $\sqrt{\frac{P}{M}} - h$ as some candidates wrongly attempted to take the square root of the expression. A common error in the first move was to subtract M and write $P - M = g^2 + h^2$. Many thought that $\sqrt{g^2 + h^2}$ was $g + h$. Those who chose to expand the brackets first usually wrote $= Mg^2 + Mh^2$ and then most of them gave the correct answer in a slightly different form. Square roots were mostly written correctly by including the whole fraction.

Question 12

Those who did not gain full marks to this question usually showed the working $\left(\frac{11}{12} + \frac{9}{12}\right)$ and then wrote their answer as an improper fraction (usually $\frac{5}{3}$). Some chose other denominators such as 48 and this did not impede them gaining full marks.

Question 13

Nearly all candidates got this correct with an answer of 1.6×10^{-3}, one of the few errors seen was 1.6×10^3 and in these cases candidates usually achieved the mark for 0.0016 seen in their working.

Question 14

(a) This was usually answered correctly, the only incorrect answer seen was 243.

(b) Some candidates did not multiply the brackets out accurately whilst others gave p the value 4^2 or 16 and q the value 5.

Question 15

Candidates needed to do 84 ÷ 1.2 but many attempted to do 84 − 84 × 0.2.

Question 16

The most common correct method was to form two equations and solve one, then substitute into the other i.e. $J+M=26$ and $J=5+M$, then substitute the second into first to produce $5+M+M=26$, giving $M=10.5$ and $J=15.5$. Candidates then doubled their values to produce the correct ratio. Some candidates did not convert 10.5 and 15.5 to 21 and 31, or they presented the answer the wrong way around as 21:31. The most common error was producing 9:4 from 18:8, which came from $26\div 2=13$ and then adding and subtracting 5 to reach 18 and 8.

Question 17

There were few good attempts at this question as most candidates did not know how to calculate the number of sides.

Question 18

The most common method here was work out the gradient $\frac{10--2}{3-2}=12$, then substitute a point to produce c = −26. The most common error was to invert the gradient formula and produce $\boldsymbol{m}=\frac{1}{12}$, or to draw the diagram and achieving the correct gradient but then the incorrect *y* intercept value.

Question 19

Few candidates could answer this question correctly. Some could write the arc length as $\frac{120}{360}\times 2\times\pi\times 8$, or similar, but most of them could not work out the side length of the triangle using trigonometry.

Question 20

Few candidates knew how to do this and most failed to draw the line *OP*.

Question 21

This was often well answered. Common incorrect answers were $243y^6$ or 27^6

Question 22

Most candidates treated this a direct proportion to the square of $x + 1$, and so obtained an answer of 169 from $(12 + 1)^2$.

Question 23

Some candidates achieved 18 with some power of *x*, whilst others attempted the highest common factor and gave 3 or 3*x*.

Question 24

(a) A few candidates answered correctly and some did manage to get $6^2 + 2^2$, or similar, but could get no further.

(b) Many candidates did draw a right-angled triangle but they then put the angle of 30° in the wrong place at the top. The use of the sine rule was unnecessary, as it could be worked out simplest using the tan ratio.

Question 25

The most common method for this question was in three steps as follows $\dfrac{3x(x-6)}{a(x-6)+2c(x-6)}$ then $\dfrac{3x(x-6)}{(a+2c)(x-6)}$ and finally $\dfrac{3x}{a+2c}$. The most common error was to cancel one of the $(x - 6)$ factors from the denominator in the first step above producing:- $\dfrac{3x}{a(x-6)+2c}$ or $\dfrac{3x}{a+2c(x-6)}$ with a final answer of $\dfrac{3x}{(a+2c)(x-6)}$.

Question 26

(a) A very few knew how to find this vector, most wrote $\boldsymbol{tr}$ or $\boldsymbol{r}^2 + \boldsymbol{t}^2$.

(b) Most of the responses halved the answer to **part (a)** and $(\boldsymbol{t} + \boldsymbol{r}) \div 2$ was common.

Question 27

There were very few answers that were correct or nearly correct. Here they attempted to undo the root first, often seen as the cube root of 2 and then the negative power was dealt with by making their answer negative.

MATHEMATICS (US)

Paper 0444/43
Paper 43 (Extended)

Key messages

To do well in this paper candidates need to be familiar with all aspects of the syllabus. The recall and application of formulae in varying situations is required, as well as the ability to interpret situations mathematically and problem solve with unstructured questions. Work should be clearly and concisely expressed with intermediate values written to at least four significant figures with only the final answer rounded to the appropriate level of accuracy. Candidates should show full working with their answers to ensure method marks are considered when final answers are incorrect.

General comments

There were some scripts in which candidates demonstrated an expertise with the majority of the content. However, there were some poorer scripts in which a lack of expertise was clearly evident, and also a lack of familiarity with some topics resulting in high numbers of no responses. There was no evidence that candidates were short of time, as most candidates attempted some or all of the later questions. Candidates should be encouraged to read questions again once they have reached a solution. This would help to avoid unnecessary loss of marks, such as in **Question 3(a)**, where several gave the total balance in both parts rather than the interest paid. Candidates should not make assumptions about geometrical diagrams, for example, in **Question 2(a)**, assuming the angle at the centre was a right angle. The topics that proved to be more accessible were simple transformations, simple interest, mass, volume and surface area of a prism, straightforward algebraic manipulation, simultaneous equations, pie charts, simple probability and some of the early work on functions. The more challenging topics were angles and cyclic quadrilaterals, reverse exponential decrease, the sine and cosine rules, area of a quadrilateral, shortest distance, setting up equations and their solution, mean of a grouped data, identifying curves from sketch graphs, harder probability and harder function questions.

Comments on specific questions

Section A

Question 1

(a) Almost all candidates attempted a description of the transformation with varying degrees of success. The stronger candidates displayed a good understanding of rotation and almost always gave a correct description. Weaker candidates tended to omit at least one aspect of the rotation, the centre, the direction of rotation or the angle of rotation. A significant number of responses included a second transformation, usually a translation of $\begin{pmatrix}1\\1\end{pmatrix}$ or $\begin{pmatrix}1\\-1\end{pmatrix}$.

(b) (i) A large minority of the responses gave a correct description of the transformation. Other responses included a variety of errors. Describing the transformation as a move, movement or translocation was fairly common, all of which were not acceptable. There were many errors involving the vector with some giving coordinates instead of a vector, some vectors included the correct figures but some were reversed or the signs were incorrect. A significant number of responses ignored the use of a vector and gave a worded description instead.

(ii) Calculating the distance moved by each point of the triangle proved challenging for a majority of candidates. Most that attempted Pythagoras were usually successful, with errors usually resulting from forgetting to take the square root or multiplying the result by 3 for the three points. By far the most common error was giving an answer of 5, simply adding the horizontal and vertical displacements together.

Question 2

(a) A significant number of candidates made little progress in this part. Many were able to identify angle *AOB* as 104° but many did not recognise that this was an example of the angle at the centre and that angle *APB* was the corresponding angle at the circumference. Common errors included assumptions that angle *ABP* was a right angle and that angle *AOB* was a right angle. Several candidates gave their answer as 38°.

(b) (i) Only the stronger candidates seemed familiar with opposite angles of a cyclic quadrilateral adding up to 180° and angle *CDE* = 80 was a common error. Others resorted to labelling as many angles as possible on the diagram, without success, as they also seemed unfamiliar with angles in the same segment.

(ii) Finding the two base angles of the isosceles triangle was a common starting point, but unfamiliarity with the rule for angles in the same segment was the downfall of many candidates and fully correct answers were in the minority. Some candidates resorted to using a protractor to measure the angle.

Question 3

(a) (i) A majority of responses calculated the correct simple interest on the investment. Incorrect responses usually involved calculating the total value of the investment rather than the simple interest, calculating the interest for just one year, or calculating the compound interest on the investment.

(ii) Some candidates had a good understanding of compound interest. Unfortunately, a significant number stopped after calculating the value of the investment and fully correct answers were in the minority. Some candidates seemed unfamiliar with the formula for compound interest and calculations such as $(500\times0.03)^7$ and $500(0.03)^7$ were often seen.

(b) Appreciating the similarity between depreciation and compound interest usually meant that candidates found the correct value of the car. Those that started with $6269.4 = A(1 - 0.1)^3$ usually went on to calculate the required value. Many of the incorrect answers involved 6269.4 being multiplied or divided by numbers such as 1.1^3, 0.9^3, 0.7^3 and several others of a similar nature. Many of these calculations gave answers less than 6269.4 and candidates seemed unaware that these could not be correct answers.

Question 4

(a) (i) Some candidates had no difficulty in calculating the mass of the prism. Errors usually involved giving the answer in grams, using an incorrect conversion from grams to kilograms or dividing the volume by the density.

(ii) Success in this part depended on knowing the relationship between the volume and the area of the cross-section and also knowing the correct method for finding the area of the cross-section. Fully correct solutions were in the majority with most incorrect answers resulting from errors in finding the area. Multiplying 10×5 was the most common error.

(iii) Calculating the total surface area proved more challenging. If the correct length of *BD* was obtained then candidates usually went on to find the correct area. Most opted to use Pythagoras' Theorem to find *BD* but not all used the correct length of 3 cm while others just assumed *BD* was 5 cm. When *BD* was correct, errors usually involved the omission of one or more faces.

(iv) Only a minority of candidates showed an understanding of the relationship between a linear scale factor and a volume scale factor. Those that used the factor of $\left(\frac{10}{5}\right)^3$ almost always reached the correct answer. Most incorrect answers resulted from the use of a scale factor of 2.

(b) Most candidates had some understanding of population density and were able to calculate the area of the town. Having found the correct area several struggled to find the radius, often using an incorrect formula or incorrectly manipulating πr^2, usually involving the square root.

Question 5

(a) Most candidates produced some good algebra and solved the simultaneous equations correctly with the elimination method being used more often than a substitution method. Incorrect answers often resulted from slips with the signs, slips with arithmetic or inconsistent addition/subtraction of their two equations. Following errors in finding the value of their first variable, some were successful in finding a value for the second variable and have a pair that satisfied one of the equations.

(b) Fewer fully correct solutions were seen in this part. Some responses demonstrated good algebraic skills but, all too often, candidates were unable to deal with the fractions correctly. This led to working such as $x + 2x = 12$. Reaching $11x = 12$ and giving the final answer as $\frac{11}{12}$ was another common error.

(c) (i) Candidates had a better understanding of inequalities and more fully correct solutions were seen. Those responses that started by separating the statement into two inequalities were generally more successful than those that opted to work with the statement in its entirety. Starting from $-8 < 3x - 2 \leq 7$ and rearranging to statements such as $3x + 6 \leq 15$ or $3x - 2 > -15$ were common errors, as well as slips with the signs or slips with the arithmetic.

(ii) Not all of those candidates with a correct solution to the inequality gave a correct list of integers. Common errors usually involved the omission of 0, the inclusion of -2, the exclusion of 3 or just giving one possible integer. Some of those with an incorrect answer to the previous part were able to solve their inequality correctly.

(d) A correct factorisation was seen in many of the responses. Common errors included partial factorisation, usually by taking 4 or a as the common factor, or attempts to treat it as the difference of two squares leading to answers such as $(4 - 2a)(4 + 2a)$.

(e) (i) Most candidates dealt with the division correctly, inverting $\frac{3}{4b}$ and multiplying. For these candidates, the most common error was forgetting to cancel either before or after the multiplication. Some attempted to write both fractions with a common denominator, usually $8ab$, before multiplication but this often resulted in incomplete cancellation. Other errors involved inverting both fractions.

(ii) Stronger candidates had no difficulty in writing the two terms with a common denominator and simplifying. Having reached the correct fraction of $\frac{x-2}{x-1}$, some spoiled their answer by incorrectly cancelling the x terms. Other errors included incorrect multiplication of $2(x - 1)$ as $2x - 1$, working with the numerator only and slips with the signs.

Question 6

(a) Many candidates seemed unfamiliar with the sine rule and fully correct solutions were rare. In some responses, angle *ADC* was treated as 90° even though clearly labelled as 100°. Responses such as 12sin50° and 12 cos30° were relatively common.

(b) Candidates fared no better in this part. It was clear from most responses that candidates were unfamiliar with the cosine rule. Most errors involved the use of simple trigonometry with candidates assuming that angle *ABC* was 90°. A higher proportion of candidates made no attempt at a response.

(c) Again, a lack of familiarity with area and trigonometry minimised any success. Very few candidates attempted to use $\frac{1}{2}ab\sin C$, often using $\frac{b\times h}{2}$ instead with no attempt to find the perpendicular height. Fully correct answers were rare as success was often dependent on previous working. In addition, premature rounding in previous parts as well as this part also affected the accuracy of the final answer. A greater proportion of candidates made no attempt at a response.

(d) Some candidates were able to identify the shortest distance by drawing the appropriate perpendicular height on the diagram. For most this was the limit of their success as many applied the wrong trigonometry ratio for their diagram. Some thought that the line from *B* to the midpoint of *AC* was the shortest distance. In a small number of cases, calculating angle *ACB* using the cosine rule followed by simple trigonometry was seen.

Question 7

(a) This proved challenging and fully correct solutions were in a minority. When a valid equation was set up it was almost always solved correctly. Writing a correct expression for the total cost, in cents, of three cakes and two loaves was common but, more often than not, this was equated to the total cost in dollars. Forming an expression for the total cost of three cakes and one loaf and equating this to the given total was another common error.

(b) A greater number of correct solutions were seen in this part. Candidates that set up a valid equation or pair of equations usually succeeded in finding the cost of a bottle of water. Stating the equation $22w = 42(w + 1)$ was a common error. Some realised that the difference of $20 in the total cost meant that 20 bottles of each drink were bought and simply divided 22 by 20.

Question 8

(a) (i) Candidates were more successful in this part and correct responses were seen. Common errors included the calculation 600 ÷ 60 and gave the answer as 100 while some used a completed grid method but did not show the required calculation.

(ii) Candidates were far more successful in this part and a large majority calculated the correct number of people.

(iii) Many correct pie charts were seen. Some candidates calculated the correct sector angle but were unable to draw it accurately or drew it freehand.

(b) Only a minority of the responses had correct box-and-whisker plots. Some candidates had all the correct figures but a lack of accuracy when drawing some of the lines, sometimes without a ruler, led to an incorrect plot. Plotting 4.1 and 8.1 without doing the necessary calculations was a common error.

(c) (i) Only the better candidates gave a correct value for the range of the scores. Many candidates worked with the frequencies as the data values, giving the range as 17. This was by far the most common error.

(ii) As in the previous part, many candidates worked with the frequencies as the data values. Consequently, fewer correct answers were seen. Common errors included 25, the frequency of the mode, and answers such as 0 or no mode while some calculated the mean of the frequencies.

(iii) Stronger candidates had no difficulty in finding the median. Common errors included finding the median of the frequencies, ordering the scores based on frequency and finding the median from this order, and in a few cases the mean was calculated.

(d) Candidates seemed unfamiliar with the topic of mean of a frequency distribution and fully correct answers were rare. Few candidates attempted to find the midpoints of the class intervals with incorrect methods usually involving the interval widths instead of the midpoints or, to a lesser extent, finding the mean of the frequencies or the mean of the midpoints.

Question 9

(a) (i) Although this proved challenging for some candidates, a majority gave the correct coordinates of the three points. Some did not know how to start, some attempted to expand the brackets but got nowhere, some attempted a variety of substitutions, and in some cases the *y*-coordinates were values other than 0.

(ii) A high proportion of candidates identified the correct sketch graph.

(b) This proved very demanding and correct solutions were extremely rare. Common errors included linear expressions or expressions involving both *x* and *y*. A higher proportion of candidates made no attempt at a response.

(c) (i) Stronger candidates had no difficulty in giving the correct range for the given function. Most of the other candidates gave the domain of the function or the values of *x* at the intercepts on the *x*-axis. A high proportion of candidates made no attempt at a response.

(ii) Candidates were more successful in finding the value of *a* than *b*. Common errors usually involved giving two of the values of *x* labelled on the axis. A high proportion of candidates made no attempt at a response.

(d) (i) Many candidates realised that the transformation was a translation but failed to use the correct terminology, often just giving simple answers such as 'right 5 units'. Almost all candidates who realised it was a translation opted to describe the displacement in words rather than use a column vector.

(ii) Those that attempted the question gave a variety of transformations, with more opting for a stretch than any other. If stretch was given it was always accompanied by the correct scale factor. Defining the invariant line was almost always missing. The most common error was defining the transformation as a translation.

Question 10

(a) (i) All candidates were able to identify the most likely number.

(ii) Only a minority of candidates found the correct probability. Some candidates seemed to ignore the fact that the spinner had two 1s and $\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$ was a common incorrect answer. The answer $\frac{1}{2}$ was another common wrong answer.

(iii) This proved challenging and correct values for *n* were rare. Some candidates attempted to use trial and improvement to find a value of *n* that satisfied $\left(\frac{3}{4}\right)^{n-1}\times\frac{1}{4}=\frac{729}{16384}$. Others worked with either the denominator or the numerator only and attempted to solve $4^n = 16384$ or $3^x = 729$. With the latter, some forgot to add on one after finding $x = 6$. Many of the incorrect attempts involved multiplying or dividing the given probability by $\frac{1}{4}$ or simply converting the given probability to a decimal.

(b) (i) Many correct responses were seen. Weaker candidates either gave the mean of the two given probabilities or the sum of the two probabilities as their answer.

(ii) This proved to be more demanding and only a small minority found the two possible combinations and added them to find the correct probability. Several found only one of the two combinations and gave that as their answer. Others found both but multiplied them or subtracted them rather than added them. The option of 1 – (passing both tests + passing neither test) was not seen. Candidates made no attempt at drawing a probability tree.

Question 11

(a) (i) All candidates found the correct value.

(ii) Although candidates were less successful in this part many did find the correct value. Most errors were the result of slips with the numeracy.

(b) This proved more challenging and fully correct solutions were in the minority. Success depended on a correct interpretation of gf(x) and its correct manipulation. Writing gf(x) as $(2x-1)^2+2(2x-1)$ was frequently seen as were $(2x-1)^2+2x$ and $(2x-1)\left(x^2+2x\right)$. Errors in expanding $(2x-1)^2$ were common with most involving the omission of the x term, or squaring the x term as $2x^2$. When expanded correctly most candidates went on to solve the equation correctly. A small number opted to factorise the original expression as $(2x-1)(2x-1+2)$ and usually went on to solve the equation correctly.

(c) Few candidates recognised p(x) as the inverse of f(x) and those that did usually started correctly but made errors in manipulating the algebra, usually involving an incorrect sign. Some simply gave $2x - 1$ as their final answer and a few answers of $\frac{1}{2x-1}$ were seen.

(d) This proved to be the most demanding question on the paper and fully correct solutions were rarely seen. It was common to see the product $4^x \cdot 2^x$ but very few were able to proceed correctly. The product was often written as one of 8^{2x}, 8^{x^2}, and to a lesser extent as 6^{2x}. After setting up a correct equation some were unsure what to do with the square root of 2. Those that rewrote it as a power of 2 usually went on to find the value of x. Some opted to change it to a decimal but this did not lead to a correct value of x.

Question 12

(a) This proved challenging for almost all candidates. Many did not read the question carefully enough and attempted to solve the quadratic equation rather than derive it. Some candidates found expressions for the times of the separate parts of the journey by calculating time as distance times speed leading to $9x + 5(2x + 1) = 2.5$.

(b) Many found the question too demanding and correct answers were rarely seen. Those that solved the equation in the previous part did not use their answer to obtain the running speed preferring instead to generate expressions but without much success. Some did not refer back to the information at the start of the previous part and attempted to give the walking speed as their answer instead of the running speed. A significant proportion of candidates made no attempt at a response.