



Cambridge International AS & A Level

THINKING SKILLS

9694/33

Paper 3 Problem Analysis and Solution

May/June 2020

2 hours



You must answer on the enclosed answer booklet.

You will need: Answer booklet (enclosed)
Calculator

INSTRUCTIONS

- Answer **all** questions.
- Follow the instructions on the front cover of the answer booklet. If you need additional answer paper, ask the invigilator for a continuation booklet.
- You should use a calculator where appropriate.
- Show your working.
Where a final answer is incorrect or missing, you may still be awarded marks for correct steps towards a solution.
In most questions, full marks will be awarded for a correct answer without any working. In some questions, however, you will not be awarded full marks if working needed to support an answer is not shown.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages. Blank pages are indicated.

1 In Bolandian currency, \$1 is worth 100 cents (ϕ). Only the following coins are used:

1 ϕ , 2 ϕ , 5 ϕ , 10 ϕ , 20 ϕ and 50 ϕ

On Monday, Peter has \$1.10 in his pocket, all in coins, but he cannot make exactly \$1 using these.

(a) How many of which coins must Peter have in his pocket on Monday? [1]

On Tuesday, Peter has some coins in his pocket, but he still cannot make exactly \$1 using these.

(b) What is the maximum amount of money that Peter could have in his pocket on Tuesday? Write down the coins that he has. [2]

On Wednesday, Peter decides that he wants to be able to make any amount of money in cents up to and including \$1.60.

(c) What is the smallest possible total number of coins which will enable him to do this? Write down the coins that he would need. [2]

On Thursday, Peter decides to use his new purse. His new purse can hold up to 12 coins in total, of any values. He wants to be able to make any amount of money in cents up to the maximum possible value.

(d) What is this maximum value? [2]

On Friday, Peter leaves his purse at home and keeps his coins in his pocket again. He decides that, as on Wednesday, he wants to be able to make any amount of money in cents up to and including \$1.60. This time, however, he does not mind how many coins he needs to have in his pocket, but he wants the total **weight** of the coins to be as small as possible.

The weights, in grams, of the different coins are shown in the table below.

Value (ϕ)	Weight (g)
1	5
2	5
5	5
10	10
20	5
50	20

(e) What is the smallest possible weight of coins that will allow Peter to do this? Write down a suitable set of coins. [2]

The Bolandian treasury considers introducing a 25 ϕ coin with a weight of 5g.

(f) Would this allow Peter to reduce the total weight of coins in his pocket on Friday? Explain your answer. [1]

[Question 2 begins on the next page]

- 2 Every year at the two-day Chevalier Horse Show teams of four riders from Frogford, Hockingham and Witherston Horse Clubs take part in a jumping competition. The competition consists of five rounds, all over the same course. The first four rounds take place on the first day of the show and the final round is on the second day. In each round the placings are decided by a combination of the time taken to complete the course and any penalties for hitting fences. It is not possible for two or more riders to be placed jointly in the same position in any round. Points are awarded as follows:

<i>Position</i>	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
<i>Points</i>	20	15	12	10	8	6	4	2	1

In the first and final rounds the riders all ride their own horses. However, for the second, third and fourth rounds a draw is made to allocate the twelve horses to the riders. The draw is organised such that no rider is allocated a horse from their own club in these rounds and every rider is allocated a different horse in each of the three rounds.

The final round of this year's competition is in progress. Yesterday's results are detailed below.

Frogford

<i>Rider</i>	<i>Round 1</i>		<i>Round 2</i>		<i>Round 3</i>		<i>Round 4</i>		<i>Total</i>
	<i>Horse</i>	<i>Points</i>	<i>Horse</i>	<i>Points</i>	<i>Horse</i>	<i>Points</i>	<i>Horse</i>	<i>Points</i>	
Jenny	Rocket	0	Aspen	20	Deister	0	Tamino	0	20
Mahela	Biscuit	10	Sapphire	0	Aspen	4	Meteor	10	24
Natalie	Pedro	12	Tamino	8	Verdi	20	Calypso	1	41
Robert	Norton	0	Calypso	4	Harvey	0	Deister	12	16
Team total									101

Hockingham

<i>Rider</i>	<i>Round 1</i>		<i>Round 2</i>		<i>Round 3</i>		<i>Round 4</i>		<i>Total</i>
	<i>Horse</i>	<i>Points</i>	<i>Horse</i>	<i>Points</i>	<i>Horse</i>	<i>Points</i>	<i>Horse</i>	<i>Points</i>	
Andrew	Verdi	6	Rocket	6	Meteor		Pedro	0	13
Dilani	Tamino	20	Norton	1	Calypso		Sapphire	8	41
Graham	Aspen	0	Meteor	15	Sapphire		Harvey	20	41
Sana	Deister	4	Harvey	0	Rocket		Biscuit	0	19
Team total									114

Witherston

<i>Rider</i>	<i>Round 1</i>		<i>Round 2</i>		<i>Round 3</i>		<i>Round 4</i>		<i>Total</i>
	<i>Horse</i>	<i>Points</i>	<i>Horse</i>	<i>Points</i>	<i>Horse</i>	<i>Points</i>	<i>Horse</i>	<i>Points</i>	
Brian	Sapphire	1	Biscuit	0	Pedro	2	Verdi	15	18
Hanif	Calypso	15	Pedro	10	Tamino	10	Rocket	6	41
Laura	Harvey	8	Deister	2	Norton	8	Aspen	2	20
Tamsin	Meteor	2	Verdi	12	Biscuit	0	Norton	4	18
Team total									97

- (a) All four of the Hockingham riders scored points in the third round, but they are missing from the table.

In which positions were each of the four Hockingham riders placed in the third round? [2]

- (b) Andrew is disappointed to be in last place individually after the fourth round, but he is proud of his horse Verdi.

How many points have riders from the other two clubs scored in total while riding Verdi? [1]

- (c) Which horse failed to provide its riders with any points at all in the second, third and fourth rounds? [1]

- (d) Which of the Hockingham riders rode three horses from the same club in the second, third and fourth rounds? [1]

- (e) In the second round, Sana was originally placed third. However, she was later disqualified when it was discovered that she had crossed the start line before the starting bell had been rung.

How many more points would Hockingham have scored in the second round if Sana had not been disqualified? [2]

There is a trophy for the winning team, and also one for the top individual rider. If there is a tie for first place after the fifth round, for either the team or the individual trophy, then that trophy is shared. Last year all three teams shared the team trophy. The individual trophy was also shared, between two riders.

- (f) Explain why there will definitely not be a tie for the trophy for the top individual rider this year, assuming no disqualifications. [4]

In today's final round, four riders have already jumped and the current positions in this round are as follows:

1st Tamsin; 2nd Andrew; 3rd Robert; 4th Brian

- (g) Give a final order of positions of the riders in today's round that would result in a three-way tie for the team trophy again this year. [4]

- 3 Tridaw is a game played between 2 teams of 3 players. Each round of a match is played by one player from each of the teams. A match consists of 9 rounds, divided into 3 groups of 3. Each player of a team must play in one round in each of the groups, and no two rounds can be played by the same two players against each other.

The winning team for each round in the first group scores 1 point.

The winning team for each round in the second group scores 3 points.

The winning team for each round in the third group scores 5 points.

The team with the most points at the end of the 9 rounds wins the match.

- (a) What is the lowest possible winning score for a match of Tridaw? [1]

To determine in which group each pair of players will compete against each other, the team captains take it in turns to fill in a table. For today's match between the Hawks and the Griffins, the captain of the Hawks decided to pair Karl with Steven in group 3. The captain of the Griffins then chose to pair Roger with Len in group 1. The table now looks as shown below.

		<i>Hawks</i>		
		<i>Jack</i>	<i>Karl</i>	<i>Len</i>
<i>Griffins</i>	<i>Roger</i>			1
	<i>Steven</i>		3	
	<i>Tom</i>			

There is now only one possible way in which the remaining values in the table can be completed according to the rules.

- (b) Show how all of the remaining pairs will be allocated to groups 1, 2 or 3. [2]

- (c) Give an example of an allocation of two initial pairs that would have left more than one way for the grid to be completed. [1]

The winners of each of the rounds are shown in the table below.

		<i>Hawks</i>		
		<i>Jack</i>	<i>Karl</i>	<i>Len</i>
<i>Griffins</i>	<i>Roger</i>	Roger	Karl	Roger
	<i>Steven</i>	Jack	Steven	Len
	<i>Tom</i>	Jack	Tom	Len

- (d) What was the final score in the match? [2]

After the match, Tom complained that they had lost because the captain had made the wrong decision when he chose to pair Roger with Len in group 1 (after the opposing captain had decided to pair Karl with Steven in group 3).

Tom says that, assuming that the winner of the round between any pair of players would have been the same whichever group that round was played in, the outcome of the match could have been different.

- (e) (i) What is the greatest score that Tom thinks the Griffins team could have achieved if the captain had made a different decision? [2]
- (ii) Which group would the captain have had to specify for the round between Roger and Len in order to be sure to achieve the greatest score? Explain why it is the only possibility that guarantees this greatest score. [2]

- 4 Sally is organising a charity concert next month. She has booked the hall and is deciding on the price that she should set for tickets. She has researched the numbers of tickets sold for five similar concerts in the past. In each of the concerts that she has researched only one price of ticket was available. The results of her research are shown in the table.

<i>Ticket price (\$)</i>	<i>Number available</i>	<i>Number sold</i>
20	200	161
25	200	129
30	100	100
35	150	101
40	150	79

Sally thinks that the number of people willing to buy a ticket will reduce by a similar amount for every extra \$5 charged. She assumes that anyone who attended one of the concerts would have been interested in attending all of the concerts.

- (a) Which row of the table is inconsistent with the others, if Sally is correct? [1]

For all of the concerts the profit was given to charity. The total cost for use of a venue depends on the size of the venue, which is indicated by the number of tickets available for the concert. The total cost for a venue is \$4 for each available ticket, plus an additional \$200.

- (b) Which was the concert that gave the most to charity, and how much was given? [3]

Sally devises a model to help her to plan her concert. She assumes that 160 people would be willing to buy a ticket priced at \$20, and the number will reduce by 20 people for every additional \$5 on the price.

Sally will only consider prices that are a multiple of \$5.
The maximum number of tickets that Sally can sell for her concert is 150.

- (c) What is the maximum income Sally could receive from the sale of tickets? [2]

Sally discussed her plans for the concert with a friend, Julia. There are 60 premium seats in the hall, so Julia suggested that Sally should charge a higher price for these. The other 90 seats would be offered at the standard price.

Sally makes the following assumptions:

- A) Any customer who is willing to pay for one will buy a higher-priced ticket, if one is still available.
- B) Any customer who is willing to buy a higher-priced ticket is willing to buy one of the standard tickets, if no higher-priced tickets are available.

For example, if the higher price were \$25 and the standard price \$20, all 60 premium tickets would be sold and there would still be 100 people willing to buy a standard ticket, so all 150 tickets would be sold.

- (d) What two prices should Sally set for the tickets to give the maximum possible income? [3]

Julia pointed out to Sally that she disagrees with the first of Sally's assumptions (assumption A). She thinks that some of the customers who would be willing to buy tickets at the higher price might still choose to buy the standard ones if they are still available. Julia estimates that half of the customers who would buy at the higher price would do this.

Sally therefore adjusts her model so that the number of people who would be willing to buy a higher-priced ticket is now half of what it was before.

- (e) (i) By how much would Sally expect her income to be reduced, if she keeps the prices found in part (d)? [2]
- (ii) With the adjusted model, what two prices will produce the highest possible income? [4]

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