



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/41

Paper 4 Further Probability & Statistics

October/November 2024

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.



- 1 Ellie is investigating the heights of two types of beech tree, A and B , in a certain region. She has chosen a random sample of 60 beech trees of type A in the region, recorded their heights, x m, and calculated unbiased estimates for the population mean and population variance as 35.6 m and 4.95 m^2 respectively.

Ellie also chooses a random sample of 50 beech trees of type *B* in the region and records their heights, ym. Her results are summarised as follows.

$$\Sigma y = 1654 \quad \Sigma y^2 = 54\,850$$

Find a 95% confidence interval for the difference between the population mean heights of type *A* and type *B* beech trees in the region. [6]

[illegible]

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3 A statistician believes that the number of telephone calls received by an advice centre in a 10-minute interval can be modelled by the Poisson distribution $\text{Po}(1.9)$. The number of calls received in a randomly chosen 10-minute interval was recorded on each of 100 days. The results are summarised in the table, together with some of the expected frequencies corresponding to the distribution $\text{Po}(1.9)$.

Number of calls	0	1	2	3	4	5	6 or more
Observed frequency	10	18	35	21	11	4	1
Expected frequency	14.957	28.418	26.997				1.322

(a) Complete the table. [2]

[illegible]

(b) Carry out a goodness of fit test, at the 10% significance level, to determine whether the statistician's belief is reasonable. [6]

[illegible]



This image shows a full page of a handwriting practice sheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.





- 4 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} kx^3 & 0 \leq x < 1, \\ k(5-x) & 1 \leq x \leq 5, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

- (a) Sketch the graph of f .

[1]

- (b) Show that $k = \frac{4}{33}$.

[2]

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(c) Find the cumulative distribution function of X .

[3]

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(d) Find the median value of X .

[4]

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- 5 Nikita has three coins. One coin is fair, one coin is biased so that the probability of obtaining a head is $\frac{1}{3}$ and the third coin is biased so that the probability of obtaining a head is $\frac{1}{5}$. The random variable X is the number of heads that Nikita obtains when he throws all three coins at the same time.

(a) Find the probability generating function of X . [3]

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Rajesh has two fair six-sided dice with faces labelled 1, 2, 3, 4, 5, 6. The random variable Y is the number of 4s that Rajesh obtains when he throws the two dice.

The random variable Z is the sum of the number of heads obtained by Nikita and the number of 4s obtained by Rajesh.

(b) Find the probability generating function of Z , expressing your answer as a polynomial. [4]

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(c) Use your answer to part (b) to find $E(Z)$. [2]

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