



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/41

Paper 4 Further Probability & Statistics

October/November 2022

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

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- 2 An organisation runs courses to train students to become engineers. These students are taught in groups of 8. The director of the organisation claims that on average 60% of the students in a group achieve a pass. A random sample of 150 groups of 8 students is chosen. The following table shows the observed frequencies together with some of the expected frequencies using the appropriate binomial distribution.

| | | | | | | | | | |
|----------------------------|-----|-------|-------|--------|--------|-----|-----|--------|-------|
| Number of passes per group | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Observed frequency | 0 | 0 | 8 | 24 | 45 | 36 | 26 | 10 | 1 |
| Expected frequency | p | 1.180 | 6.193 | 18.579 | 34.836 | q | r | 13.437 | 2.519 |

- (a) Find the values of p , q and r giving your answers correct to 3 decimal places. [2]

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- (b) Carry out a goodness of fit test, at the 10% significance level, to test whether there is evidence to reject the director’s claim. [6]

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(b) Give a reason why it is preferable to use a Wilcoxon matched-pairs signed-rank test in this situation rather than a paired sample *t*-test. [1]

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4 Jason has three biased coins. For each coin the probability of obtaining a head when it is thrown is $\frac{2}{3}$. Jason throws all three coins. The number of heads obtained is denoted by X .

(a) Find the probability generating function $G_X(t)$ of X . [3]

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Jason also has two unbiased coins. He throws all five coins. The number of heads obtained from the two unbiased coins is denoted by Y . It is given that $G_Y(t) = \frac{1}{4} + \frac{1}{2}t + \frac{1}{4}t^2$. The random variable Z is the total number of heads obtained when Jason throws all five coins.

(b) Find the probability generating function of Z , expressing your answer as a polynomial. [3]

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(c) Find $E(Z)$. [2]

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The random variable Y is given by $Y = \sqrt{X}$.

(c) Find the probability density function of Y . [3]

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