Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER	

4551344185

FURTHER MATHEMATICS **9231/23**

Paper 2 **October/November 2019**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **23** printed pages and **1** blank page.

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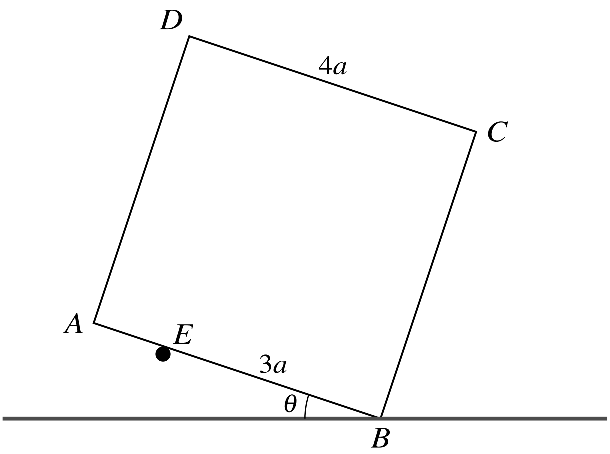
1 A particle P is moving in a circle of radius 2 m. At time t seconds, its velocity is $(t-1)^2\ \mathrm{m\,s^{-1}}$. At a particular time T seconds, where $T > 0$, the magnitude of the radial component of the acceleration of P is $8\ \mathrm{m\,s^{-2}}$. Find the magnitude of the transverse component of the acceleration of P at this instant. [5]

2



A uniform square lamina $ABCD$ of side $4a$ and weight W rests in a vertical plane with the edge AB inclined at an angle θ to the horizontal, where $\tan\theta = \frac{1}{3}$. The vertex B is in contact with a rough horizontal surface for which the coefficient of friction is μ. The lamina is supported by a smooth peg at the point E on AB, where $BE = 3a$ (see diagram).

(i) Find expressions in terms of W for the normal reaction forces at E and B. [5]

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(ii) Given that the lamina is about to slip, find the value of μ. [3]

3 Three uniform small spheres A, B and C have equal radii and masses $5m$, $5m$ and $3m$ respectively. The spheres are at rest on a smooth horizontal surface, in a straight line, with B between A and C. The coefficient of restitution between each pair of spheres is e. Sphere A is projected directly towards B with speed u.

(i) Show that the speed of A after its collision with B is $\frac{1}{2}u(1 - e)$ and find the speed of B. [3]

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Sphere B now collides with sphere C. Subsequently there are no further collisions between any of the spheres.

(ii) Find the set of possible values of e. [6]

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4 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O and P is held with the string taut and horizontal. The particle P is projected vertically downwards with speed $\sqrt{(2ag)}$ so that it begins to move along a circular path. The string becomes slack when OP makes an angle θ with the upward vertical through O.

(i) Show that $\cos\theta = \frac{2}{3}$. [5]

(ii) Find the greatest height, above the horizontal through O, reached by P in its subsequent motion. [4]

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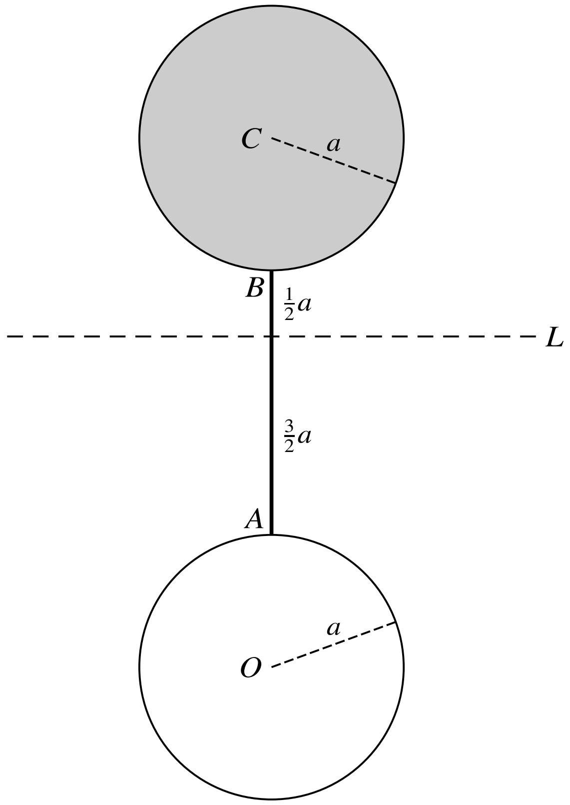

A thin uniform rod AB has mass λM and length $2a$. The end A of the rod is rigidly attached to the surface of a uniform hollow sphere (spherical shell) with centre O, mass $3M$ and radius a. The end B of the rod is rigidly attached to the surface of a uniform solid sphere with centre C, mass $5M$ and radius a. The rod lies along the line joining the centres of the spheres, so that $CBAO$ is a straight line. The horizontal axis L is perpendicular to the rod and passes through the point of the rod that is a distance $\frac{1}{2}a$ from B (see diagram). The object consisting of the rod and the two spheres can rotate freely about L.

(i) Show that the moment of inertia of the object about L is $\left(\frac{408 + 7\lambda}{12}\right)Ma^2$. [6]

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The period of small oscillations of the object about L is $5\pi\sqrt{\left(\frac{2a}{g}\right)}$.

(ii) Find the value of λ. [6]

6 A random sample of 9 members is taken from the large number of members of a sports club, and their heights are measured. The heights of all the members of the club are assumed to be normally distributed. A 95% confidence interval for the population mean height, μ metres, is calculated from the data as $1.65 \leqslant \mu \leqslant 1.85$.

(i) Find an unbiased estimate for the population variance. [3]

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(ii) Denoting the height of a member of the club by x metres, find Σx^2 for this sample of 9 members. [4]

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7 The time, T days, before an electrical component develops a fault has distribution function F given by

$$F(t) = \begin{cases} 1 - e^{-at} & t \geqslant 0, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a positive constant. The mean value of T is 200.

(i) Write down the value of a. [1]

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(ii) Find the probability that an electrical component of this type develops a fault in less than 150 days. [2]

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A piece of equipment contains n of these components, which develop faults independently of each other. The probability that, after 150 days, at least one of the n components has not developed a fault is greater than 0.99.

(iii) Find the smallest possible value of n. [4]

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8 A random sample of 8 elephants from region A is taken and their weights, x tonnes, are recorded. (1 tonne = 1000 kg.) The results are summarised as follows.

$$\Sigma x = 32.4 \qquad \Sigma x^2 = 131.82$$

A random sample of 10 elephants from region B is taken. Their weights give a sample mean of 3.78 tonnes and an unbiased variance estimate of 0.1555 tonnes^2. The distributions of the weights of elephants in regions A and B are both assumed to be normal with the same population variance. Test at the 10% significance level whether the mean weight of elephants in region A is the same as the mean weight of elephants in region B. [9]

9 A random sample of five pairs of values of x and y is taken from a bivariate distribution. The values are shown in the following table, where p and q are constants.

x	1	2	3	4	5
y	4	p	q	2	1

The equation of the regression line of y on x is $y = -0.5x + 3.5$.

(i) Find the values of p and q. [7]

(ii) Find the value of the product moment correlation coefficient. [3]

10 The random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{30}\left(\frac{8}{x^2} + 3x^2 - 14\right) & 2 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the distribution function of X. [3]

The random variable Y is defined by $Y = X^2$.

(ii) Find the probability density function of Y. [4]

(iii) Find the value of y such that $P(Y < y) = 0.8$. [3]

11 Answer only **one** of the following two alternatives.

EITHER

The points A and B are a distance 1.2 m apart on a smooth horizontal surface. A particle P of mass $\frac{2}{3}$ kg is attached to one end of a light spring of natural length 0.6 m and modulus of elasticity 10 N. The other end of the spring is attached to the point A. A second light spring, of natural length 0.4 m and modulus of elasticity 20 N, has one end attached to P and the other end attached to B.

(i) Show that when P is in equilibrium $AP = 0.75$ m. [3]

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The particle P is displaced by 0.05 m from the equilibrium position towards A and then released from rest.

(ii) Show that P performs simple harmonic motion and state the period of the motion. [6]

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(iii) Find the speed of P when it passes through the equilibrium position. [2]

(iv) Find the speed of P when its acceleration is equal to half of its maximum value. [3]

OR

The number of puncture repairs carried out each week by a small repair shop is recorded over a period of 40 weeks. The results are shown in the following table.

Number of repairs in a week	0	1	2	3	4	5	$\geqslant 6$
Number of weeks	6	15	9	6	3	1	0

(i) Calculate the mean and variance for the number of repairs in a week and comment on the possible suitability of a Poisson distribution to model the data. [3]

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Records over a longer period of time indicate that the mean number of repairs in a week is 1.6. The following table shows some of the expected frequencies, correct to 3 decimal places, for a period of 40 weeks using a Poisson distribution with mean 1.6.

Number of repairs in a week	0	1	2	3	4	5	$\geqslant 6$
Expected frequency	8.076	12.921	10.337	5.513	2.205	a	b

(ii) Show that $a = 0.706$ and find the value of the constant b. [3]

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(iii) Carry out a goodness of fit test of a Poisson distribution with mean 1.6, using a 10% significance level. [8]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.