Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

* 3 9 3 3 3 8 4 5 4 *

FURTHER MATHEMATICS **9231/13**

Paper 1 **October/November 2019**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **22** printed pages and **2** blank pages.

JC19 11_9231_13/FP

Cambridge Assessment International Education

[Turn over

1 The curve C has equation $y = x^a$ for $0 \leqslant x \leqslant 1$, where a is a positive constant. Find, in terms of a, the coordinates of the centroid of the region enclosed by C, the line $x = 1$ and the x-axis. [6]

2 It is given that $y = \ln(ax + 1)$, where a is a positive constant. Prove by mathematical induction that, for every positive integer n,

$$\frac{d^n y}{dx^n} = (-1)^{n-1} \frac{(n-1)!a^n}{(ax+1)^n}.$$ [6]

3 The integral I_n, where n is a positive integer, is defined by

$$I_n = \int_{\frac{1}{2}}^{1} x^{-n} \sin \pi x \, dx.$$

(i) Show that

$$n(n+1)I_{n+2} = 2^{n+1}n + \pi - \pi^2 I_n.$$ [5]

....

....

....

....

....

....

....

....

....

....

....

....

....

(ii) Find I_5 in terms of π and I_1. [2]

....

....

....

....

....

....

....

4 The line $y = 2x + 1$ is an asymptote of the curve C with equation

$$y = \frac{x^2 + 1}{ax + b}.$$

(i) Find the values of the constants a and b. [3]

....

....

....

....

....

....

....

(ii) State the equation of the other asymptote of C. [1]

....

....

(iii) Sketch C. [Your sketch should indicate the coordinates of any points of intersection with the y-axis. You do not need to find the coordinates of any stationary points.] [3]

5 Let $S_N = \sum_{r=1}^{N}(5r+1)(5r+6)$ and $T_N = \sum_{r=1}^{N}\frac{1}{(5r+1)(5r+6)}$.

(i) Use standard results from the List of Formulae (MF10) to show that

$$S_N = \tfrac{1}{3}N(25N^2 + 90N + 83).$$ [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(ii) Use the method of differences to express T_N in terms of N. [4]

....

....

....

....

....

(iii) Find $\lim_{N\to\infty}\left(N^{-3}S_N T_N\right)$. [2]

6 With O as the origin, the points A, B, C have position vectors

$$\mathbf{i} - \mathbf{j}, \quad 2\mathbf{i} + \mathbf{j} + 7\mathbf{k}, \quad \mathbf{i} - \mathbf{j} + \mathbf{k}$$

respectively.

(i) Find the shortest distance between the lines OC and AB. [5]

(ii) Find the cartesian equation of the plane containing the line OC and the common perpendicular of the lines OC and AB. [4]

7 The equation $x^3 + 2x^2 + x + 7 = 0$ has roots α, β, γ.

(i) Use the relation $x^2 = -7y$ to show that the equation

$$49y^3 + 14y^2 - 27y + 7 = 0$$

has roots $\frac{\alpha}{\beta\gamma}$, $\frac{\beta}{\gamma\alpha}$, $\frac{\gamma}{\alpha\beta}$. [4]

(ii) Show that $\dfrac{\alpha^2}{\beta^2\gamma^2} + \dfrac{\beta^2}{\gamma^2\alpha^2} + \dfrac{\gamma^2}{\alpha^2\beta^2} = \dfrac{58}{49}$. [3]

(iii) Find the exact value of $\dfrac{\alpha^3}{\beta^3\gamma^3} + \dfrac{\beta^3}{\gamma^3\alpha^3} + \dfrac{\gamma^3}{\alpha^3\beta^3}$. [2]

8 The matrix **M** is defined by

$$\mathbf{M} = \begin{pmatrix} 2 & m & 1 \\ 0 & m & 7 \\ 0 & 0 & 1 \end{pmatrix},$$

where $m \neq 0,\ 1,\ 2$.

(i) Find a matrix **P** and a diagonal matrix **D** such that $\mathbf{M} = \mathbf{PDP}^{-1}$. [7]

(ii) Find $\mathbf{M}^7\mathbf{P}$. [3]

9 **(i)** Use de Moivre's theorem to show that

$$\sec 6\theta = \frac{\sec^6 \theta}{32 - 48\sec^2 \theta + 18\sec^4 \theta - \sec^6 \theta}.$$ [6]

(ii) Hence obtain the roots of the equation

$$3x^6 - 36x^4 + 96x^2 - 64 = 0$$

in the form $\sec q\pi$, where q is rational. [5]

10 The matrix **A** is defined by

$$\mathbf{A} = \begin{pmatrix} 1 & 5 & 1 \\ 1 & -2 & -2 \\ 2 & 3 & \theta \end{pmatrix}.$$

(i) (a) Find the rank of **A** when $\theta \neq -1$. [3]

(b) Find the rank of **A** when $\theta = -1$. [1]

Consider the system of equations

$$x + 5y + z = -1,$$
$$x - 2y - 2z = 0,$$
$$2x + 3y + \theta z = \theta.$$

(ii) Solve the system of equations when $\theta \neq -1$. [3]

(iii) Find the general solution when $\theta = -1$. [3]

(iv) Show that if $\theta = -1$ and $\phi \neq -1$ then $\mathbf{Ax} = \begin{pmatrix} -1 \\ 0 \\ \phi \end{pmatrix}$ has no solution. [2]

11 Answer only **one** of the following two alternatives.

EITHER

It is given that $w = \cos y$ and

$$\tan y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 2\tan y \frac{dy}{dx} = 1 + e^{-2x} \sec y.$$

(i) Show that

$$\frac{d^2w}{dx^2} + 2\frac{dw}{dx} + w = -e^{-2x}.$$ [4]

....

....

....

....

....

....

....

....

....

(ii) Find the particular solution for y in terms of x, given that when $x = 0$, $y = \frac{1}{3}\pi$ and $\frac{dy}{dx} = \frac{1}{\sqrt{3}}$. [10]

....

....

....

....

....

....

....

....

OR

The curves C_1 and C_2 have polar equations, for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$, as follows:

$$C_1 : r = 2\left(e^{\theta} + e^{-\theta}\right),$$
$$C_2 : r = e^{2\theta} - e^{-2\theta}.$$

The curves intersect at the point P where $\theta = \alpha$.

(i) Show that $e^{2\alpha} - 2e^{\alpha} - 1 = 0$. Hence find the exact value of α and show that the value of r at P is $4\sqrt{2}$. [6]

(ii) Sketch C_1 and C_2 on the same diagram. [3]

(iii) Find the area of the region enclosed by C_1, C_2 and the initial line, giving your answer correct to 3 significant figures. [5]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

BLANK PAGE

BLANK PAGE