



# Cambridge International AS & A Level

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## FURTHER MATHEMATICS

9231/31

Paper 3 Further Mechanics

May/June 2023

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity ( $g$ ) is needed, use  $10 \text{ ms}^{-2}$ .

### INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1 One end of a light elastic string, of natural length  $a$  and modulus of elasticity  $3mg$ , is attached to a fixed point  $O$ . The other end of the string is attached to a particle  $P$  of mass  $m$ . The string hangs with  $P$  vertically below  $O$ . The particle  $P$  is pulled vertically downwards so that the extension of the string is  $2a$ . The particle  $P$  is then released from rest.

(a) Find the speed of  $P$  when it is at a distance  $\frac{3}{4}a$  below  $O$ . [3]

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(b) Find the initial acceleration of  $P$  when it is released from rest. [2]

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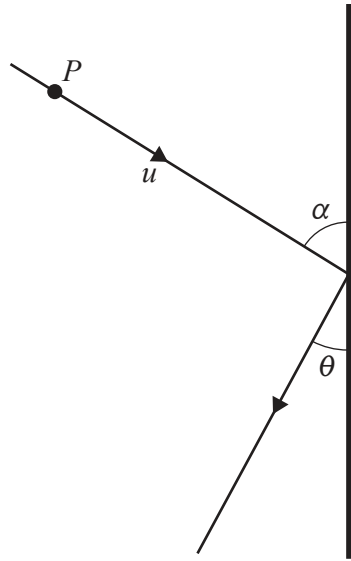
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A particle  $P$  of mass  $m$  is moving with speed  $u$  on a fixed smooth horizontal surface. It collides at an angle  $\alpha$  with a fixed smooth vertical barrier. After the collision,  $P$  moves at an angle  $\theta$  with the barrier, where  $\tan \theta = \frac{1}{2}$  (see diagram). The coefficient of restitution between  $P$  and the barrier is  $e$ . The particle  $P$  loses 20% of its kinetic energy as a result of the collision.

Find the value of  $e$ .

[5]

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3 A particle  $P$  of mass  $m$  is attached to one end of a light inextensible string of length  $a$ . The other end of the string is attached to a fixed point  $O$ . The particle  $P$  is held at the point  $A$ , where  $OA$  makes an angle  $\theta$  with the downward vertical through  $O$ , and with the string taut. The particle  $P$  is projected perpendicular to  $OA$  in an upwards direction with speed  $u$ . It then starts to move along a circular path in a vertical plane. The string goes slack when  $P$  is at  $B$ , where angle  $AOB$  is  $90^\circ$  and the speed of  $P$  is  $\sqrt{\frac{4}{5}ag}$ .

(a) Find the value of  $\sin \theta$ . [2]

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(b) Find, in terms of  $m$  and  $g$ , the tension in the string when  $P$  is at  $A$ . [5]

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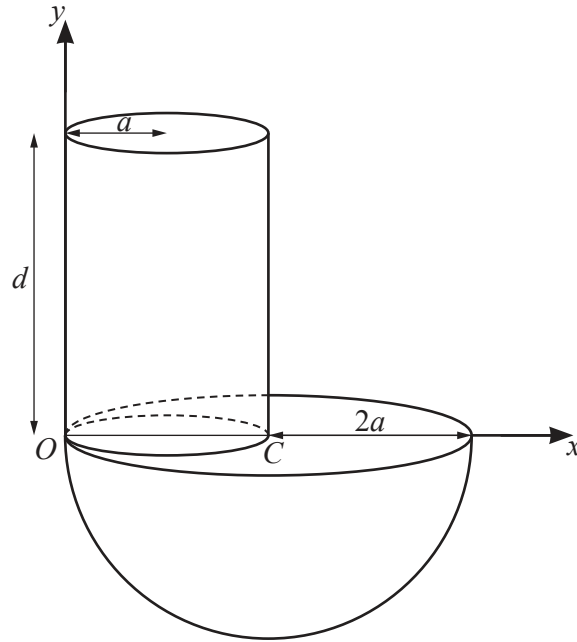
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An object is formed from a solid hemisphere, of radius  $2a$ , and a solid cylinder, of radius  $a$  and height  $d$ . The hemisphere and the cylinder are made of the same material. The cylinder is attached to the plane face of the hemisphere. The line  $OC$  forms a diameter of the base of the cylinder, where  $C$  is the centre of the plane face of the hemisphere and  $O$  is common to both circumferences (see diagram). Relative to axes through  $O$ , parallel and perpendicular to  $OC$  as shown, the centre of mass of the object is  $(\bar{x}, \bar{y})$ .

- (a) Show that  $\bar{x} = \frac{32a^2 + 3ad}{16a + 3d}$  and find an expression, in terms of  $a$  and  $d$ , for  $\bar{y}$ . [5]

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5 A light elastic string of natural length  $a$  and modulus of elasticity  $\lambda mg$  has one end attached to a fixed point  $O$  on a smooth horizontal surface. When a particle of mass  $m$  is attached to the free end of the string, it moves with speed  $v$  in a horizontal circle with centre  $O$  and radius  $x$ . When, instead, a particle of mass  $2m$  is attached to the free end of the string, this particle moves with speed  $\frac{1}{2}v$  in a horizontal circle with centre  $O$  and radius  $\frac{3}{4}x$ .

(a) Find  $x$  in terms of  $a$ . [5]

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7 At time  $t$  s, a particle  $P$  is projected with speed  $40 \text{ m s}^{-1}$  at an angle  $\theta$  above the horizontal from a point  $O$  on a horizontal plane and moves freely under gravity. The greatest height achieved by  $P$  during its flight is  $H$  m and the corresponding time is  $T$  s.

- (a) Obtain expressions for  $H$  and  $T$  in terms of  $\theta$ . [2]

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During the time between  $t = T$  and  $t = 3$ ,  $P$  descends a distance  $\frac{1}{4}H$ .

- (b) Find the value of  $\theta$ . [4]

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