Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2004404347

FURTHER MATHEMATICS **9231/21**

Paper 2 Further Pure Mathematics 2 **May/June 2023**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 **(a)** Show that the system of equations

$$x + 2y + 3z = 1,$$
$$4x + 5y + 6z = 1,$$
$$7x + 8y + 9z = 1,$$

does not have a unique solution. [2]

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(b) Show that the system of equations in part **(a)** is consistent. Interpret this situation geometrically. [3]

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2 Use the substitution $z = x + y$ to find the solution of the differential equation

$$\frac{dy}{dx} = \frac{1 + 3x + 3y}{3x + 3y - 1}$$

for which $y = 0$ when $x = 1$. Give your answer in the form $a\ln(x + y) + b(x - y) + c = 0$, where a, b and c are constants to be determined. [7]

3 **(a)** By considering the binomial expansion of $(z+z^{-1})^4$, where $z = \cos\theta + \mathrm{i}\sin\theta$, use de Moivre's theorem to show that $\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$. [5]

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(b) Use the substitution $x = \sin\theta$ to find the exact value of $\int_0^{\frac{1}{2}} (1-x^2)^{\frac{3}{2}}\,\mathrm{d}x$. [3]

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4 The integral I_n is defined by $I_n = \int_0^1 (1+x^5)^n \, dx$.

(a) By considering $\frac{d}{dx}\left(x(1+x^5)^n\right)$, or otherwise, show that

$$(5n+1)I_n = 2^n + 5nI_{n-1}. \qquad [5]$$

(b) Find the exact value of I_3. [4]

5 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 18 & 5 & -11 \\ 8 & 6 & -4 \\ 32 & 10 & -20 \end{pmatrix}.$$

(a) Show that the characteristic equation of $\mathbf{A}$ is $\lambda^3 - 4\lambda^2 - 20\lambda + 48 = 0$ and hence find the eigenvalues of $\mathbf{A}$. [4]

(b) Find a matrix **P** and a diagonal matrix **D** such that $\mathbf{A}^5 = \mathbf{PDP}^{-1}$. [6]

6 Find the particular solution of the differential equation

$$\frac{d^2x}{dt^2} - 12\frac{dx}{dt} + 36x = 37\sin t,$$

given that, when $t = 0$, $x = \frac{dx}{dt} = 0$. [11]

7 **(a)** Use the substitution $u = x^2 - 1$ to find $\int \frac{x}{\sqrt{x^2 - 1}}\,dx$. [3]

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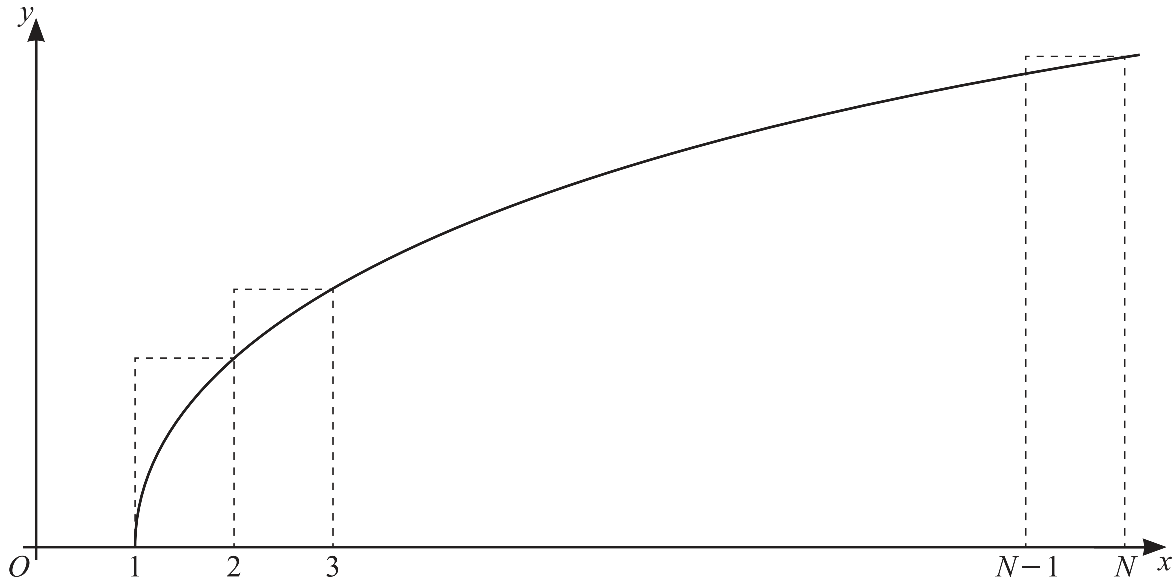

The diagram shows the curve with equation $y = \cosh^{-1} x$ together with a set of $(N-1)$ rectangles of unit width.

(b) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=2}^{N} \ln\left(r + \sqrt{r^2 - 1}\right) > N \ln\left(N + \sqrt{N^2 - 1}\right) - \sqrt{N^2 - 1}.$$ [5]

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(c) Use a similar method to find, in terms of N, an upper bound for $\sum_{r=2}^{N} \ln\left(r+\sqrt{r^2-1}\right)$. [3]

8 **(a)** Starting from the definitions of sech and tanh in terms of exponentials, prove that

$$1 - \operatorname{sech}^2 t = \tanh^2 t. \qquad [3]$$

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The curve C has parametric equations

$$x = \tfrac{1}{2}\tanh^2 t + \ln \operatorname{sech} t, \qquad y = 1 + \tanh^4 t, \qquad \text{for } t > 0.$$

(b) Show that $\frac{dy}{dx} = -4\operatorname{sech}^2 t$. [5]

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(c) Find the coordinates of the point on C with $\frac{d^2y}{dx^2} = -\frac{9}{2}$, giving your answer in the form $(a + \ln b, c)$ where a, b and c are rational numbers. [6]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.