

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9231 FURTHER MATHEMATICS

9231/21

Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Question Number	Mark Scheme Details		Part Mark	Total	
1	Equate impulse to momentum to find initial speed v and Newton's law of restitution to find new speed:	$v = 4u, v' = ev = [-] 3u$	M1 A1 2	2	
2	Find v^2 at both A and B : Find amplitude a m from given K.E. ratio: Find ω from $v_{\max} = a\omega$: Find time ($\sqrt{}$ on a) at A <i>or</i> at B , e.g.: Combine correctly to find time from A to B : Evaluate to 3 d.p.:	$v_A^2 = \omega^2(a^2 - 0.5^2)$ and $v_B^2 = \omega^2(a^2 - 0.75^2)$ $\frac{1}{2}mv_A^2 = (12/11) \frac{1}{2}mv_B^2$ $11(a^2 - 0.5^2) = 12(a^2 - 0.75^2)$ $a^2 = \frac{1}{4}(27 - 11) = 4, a = 2$ $0.6 = 2\omega, \omega = 0.3$ $\omega^{-1} \sin^{-1}(0.5/2)$ <i>or</i> $\omega^{-1} \cos^{-1}(0.5/2)$ $\omega^{-1} \sin^{-1}(0.75/2)$ <i>or</i> $\omega^{-1} \cos^{-1}(0.75/2)$ $\omega^{-1} \sin^{-1}(0.75/2) - \omega^{-1} \sin^{-1}(0.5/2)$ <i>or</i> $\omega^{-1} \cos^{-1}(0.5/2) - \omega^{-1} \cos^{-1}(0.75/2)$ $= \omega^{-1}(0.3844 - 0.2527)$ <i>or</i> $\omega^{-1}(1.318 - 1.186)$ $= 1.2813 - 0.8423$ $4.3937 - 3.9547 = 0.439$ [s]	B1 M1 A1 B1 M1 A1 M1 A1	3 5	8

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3	Use conservation of momentum, e.g.:	$mv_A + 9mv_B = mu$	M1	7	10
	Use Newton's law of restitution (consistent signs):	$v_B - v_A = eu$	M1		
	Relate v_A to v_B using K.E. (A.E.F.):	$\frac{1}{2}mv_A^2 + \frac{1}{2}9mv_B^2 = \frac{1}{2}mu^2$	M1		
	Combine two eqns to find v_A and v_B e.g.:	$v_A = (1 - 9e)u/10, v_B = (1 + e)u/10$ or $v_A, v_B = -u/2, u/6$ [or $7u/10, u/30$]	M1 A1		
	Use in 3rd eqn to find e , e.g.:	$(1 - 9e)^2 + 9(1 + e)^2 = 50$			
	(A0 if finally $\pm\frac{2}{3}$)	$90e^2 = 40, e = \frac{2}{3}$	M1 A1		
Use Newton's law of restitution with	$v_C = 2v_B', \text{ e.g.: } v_C - v_B' = ev_B, v_B' = \frac{2}{3}v_B$ $[v_B = u/6, v_B = u/9, v_C = 2u/9]$	B1	3	10	
Use conservation of momentum to find k :	$9mv_B' + kmv_C = 9mv_B$ $9v_B' + 2kv_B' = 13.5v_B', k = 9/4$	M1 A1			

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4	(i)	Use conservation of energy at lowest point: Use $F = ma$ radially at lowest point: Eliminate v^2 to find R [$v^2 = 2.3 ga$]:	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga$ $R - mg = mv^2/a$ $R = mu^2/a + 3mg = 3.3mg$	B1 B1 B1	3	10
	(ii)	Use conservation of energy at B to find V_B : (A.E.F.)	$\frac{1}{2}mV_B^2 = \frac{1}{2}mu^2 + mga \sin \theta$ $V_B^2 = (0.3 + 0.5)ga, V_B = \sqrt{(0.8ga)}$ or $2\sqrt{(ga/5)}$ or $0.894\sqrt{(ga)}$	M1A1 A1		
	(iii)	Use vertical component v_B of speed V_B at B : Find height h reached above B : Find height h reached above level of O :	$v_B = V_B \cos \theta [= \frac{1}{4}\sqrt{15} V_B = \sqrt{(3/4ga)}]$ $h = v_B^2/2g = 3a/8$ $h - a \sin \theta = 3a/8 - \frac{1}{4}a = a/8$ A.G.	M1 M1 A1 A1	4	
5	Find MI of components about A : (M1 for BC or CD)	Glass $(3M/5) \{ \frac{1}{3}(5a)^2 + 25a^2 \} = 20Ma^2$ AB $M\{ \frac{1}{3}(4a)^2 + (4a)^2 \} = 64Ma^2/3$ AD $\frac{1}{3}M\{ \frac{1}{3}(3a)^2 + (3a)^2 \} = 4Ma^2$ BC $\frac{1}{3}M\{ \frac{1}{3}(3a)^2 + 73a^2 \} = 76Ma^{2/3}$ CD $M\{ \frac{1}{3}(4a)^2 + 52a^2 \} = 172Ma^{2/3}$	M1 A1 B1 B1 M1 A1 A1	8	13	
	Find total MI about A : (OR can first find total MI about centre of mass) State or imply total mass acts at mid-point of AC	$I = 128Ma^2$ A.G.	A1 M1			
	Use eqn of circular motion to find $d^2\theta/dt^2$: Approximate $\sin \theta$ by θ and substitute for I :	$I d^2\theta/dt^2 = [-] (49Mg/15) 5a \sin \theta$ $d^2\theta/dt^2 = -(49g/384a) \theta$	M1 A1 A1			
	Find period $T = 2\pi/\omega$ with $\omega = \sqrt{(49g/384a)}$:	$T = 2\pi\sqrt{(384a/49g)}$ or $(16\pi/7)\sqrt{(6a/g)}$ or $17.6\sqrt{(a/g)}$ (A.E.F.)	B1			5

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6		State or find the expected value of X : using $p = \frac{1}{4}$:	$E(X) = 1/p = 1/\frac{1}{4} = 4$	B1	1	5
	(i)	Find $P(X = 4)$:	$P(X = 4) = (\frac{3}{4})^3 \frac{1}{4} = 27/256$ or 0.105	M1 A1	2	
	(ii)	Find $P(X < 6)$:	$P(X < 6) = 1 - (\frac{3}{4})^5$ or $\{1 + \frac{3}{4} + (\frac{3}{4})^2 + (\frac{3}{4})^3 + (\frac{3}{4})^4\} \frac{1}{4}$ $= 781/1024$ or 0.763	M1 A1	2	
		S.R. Using $p = \frac{1}{2}$ can earn B0 M1 A0 M0 A0				
7	(i)	State probability density function of T :	$f(t) = 0.001 \exp(-0.001 t) \quad (t \geq 0)$ [= 0 (otherwise or $t < 0$)]	B1	1	8
	(ii)	Find $P(T > 2000)$: S.R. $1 - e^{-2} = 0.865$ earns B1 only (max 1/3) State inequality for t (lose A1 if = or \leq): Solve for t_{\max} : (Omitting power 10 earns 0/4; using $1 - (\exp(-0.001t))^{10}$ can earn M1 A0 M1 A0 only)	$P(t > 2000) = 1 - F(2000)$ $= 1 - (1 - e^{-2}) = e^{-2}$ or 0.135 $(\exp(-0.001 t))^{10} \geq [or >] 0.9$ $t_{\max} = (\ln 0.9) / (-0.01) = 10.5$	M1 M1 A1 M1 A1 M1 A1	3 4	

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<p>8</p>	<p>State hypotheses (B0 for $\bar{\chi} \dots$):</p> <p>Estimate both popln. variances using two samples: (allow use of biased: $\sigma_{X,60}^2 = 236$ or 15.36^2)</p> <p>(allow use of biased: $\sigma_{Y,50}^2 = 265$ or 16.28^2)</p> <p>Estimate population variance for combined sample:</p> <p>(allow $\sigma_{X,60}^2/60 + \sigma_{Y,50}^2/50$: 9.233 or 3.039^2)</p> <p>Calculate value of z (to 2 d.p., either sign):</p> <p>State or use correct tabular z – value (to 2 d.p.): (or can compare 6 with e.g. 2.326 $s = 7.13$ or 7.07)</p> <p>Correct conclusion (A.E.F, \checkmark on z – values):</p> <p>S.R. Assuming equal population variances: Find pooled estimate of common variance s^2:</p> <p>Calculate value of z (to 2 d.p., either sign):</p> <p>Tabular value; conclusion</p>	<p>$H_0: \mu_X = \mu_Y, H_1: \mu_X \neq \mu_Y$</p> <p>$S_x^2 = (626220 - 6060^2/60) / 59$ [= 240 or 15.49^2]</p> <p>And $s_y^2 = (464500 - 4750^2/50) / 49$ [= 270.4 or 16.44^2]</p> <p>$s^2 = s_x^2/60 + s_y^2/50$ = 9.408 or 3.067^2</p> <p>$z = (101 - 95) / s$ = $6/3.067 = 1.96$ (or 1.97)</p> <p>$z_{0.99} = 2.326$ or 2.33 (allow 2.36)</p> <p>[Accept H_0] Claims are the same Hypotheses; Explicit assumption : $s^2 = (626220 - 6060^2/60 +$ $464500 - 4750^2/50) / 108$</p> <p>$z = 6 / s\sqrt{(1/60+1/50)} = 1.97$ = 253.8 or 15.93^2</p> <p>As above)</p>	<p>B1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1 A1 B1</p> <p>B1\checkmark (B1; B1)</p> <p>(M1 A1)</p> <p>(M1 A1) (A1)</p> <p>(B1; B1\checkmark)</p>	<p>9</p>
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<p>9</p>	<p>Find expected frequency p:</p> <p>Find q by similar method <i>or</i> by using total of 200:</p> <p>State (at least) null hypothesis: Calculate χ^2 (to 3 s.f.):</p> <p>State or use correct tabular χ^2 value (to 3 s.f.): Valid method for reaching conclusion: Conclusion consistent with correct values (A.E.F):</p>	<p>$p = 200 \int_2^3 (1/x \ln 8) dx$ $= (200 / \ln 8) [\ln x]_2^3$ $= 200 \times 0.1950 = 39.00$ A.G. $q = 21.46$ <i>or</i> 21.45</p> <p>$H_0: f(x)$ fits data (A.E.F.) $\chi^2 = 0.202 + 0.923 + 0.678 + 0.584$ $+ 1.134 + 4.134 + 3.644 = 11.3$ $\chi_{6,0.95}^2 = 12.59$ Accept H_0 if $\chi^2 \leq$ tabular value Distribution fits observations</p>	<p>M1A1 M1A1</p> <p>B1</p> <p>M1A1 B1 M1 A1</p>	<p>4</p> <p>6</p> <p>10</p>
<p>10</p>	<p>Find correlation coefficient r: (A.E.F.; A0 if only 3 s.f. clearly used)</p> <p>State both hypotheses (B0 for $r \dots$): State or use correct tabular two-tail r-value: Valid method for reaching conclusion: Correct conclusion (A.E.F, dep *A1, *B1): Calculate gradient p in $x - \bar{x} = p(y - \bar{y})$: Find regression line of x on y:</p>	<p>$r = (73\,527 - 866 \times 639 / 10) / \sqrt{\{(121\,276 - 866^2 / 10)(55\,991 - 639^2 / 10)\}}$ $= 18\,189.6 / \sqrt{(46\,280.4 \times 15\,158.9)}$ $= 0.687$</p> <p>$H_0: \rho = 0, H_1: \rho \neq 0$ $r_{10,5\%} = 0.632$ Reject H_0 if $r >$ tabular value There is non-zero correlation</p> <p>$p = 18\,189.6 / 15\,158.9 = 1.20$ $x = 86.6 + 1.20(y - 63.9)$ $= 1.20y + 9.92$</p>	<p>M1 A1 A1 *A1</p> <p>B1 *B1 M1 A1 B1 M1 A1</p>	<p>4</p> <p>4</p> <p>3</p> <p>11</p>

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11 A	(i)	Use Pythagoras to find AB : Find $\angle SAB$:	$AB = \sqrt{(4a^2 + 12a^2)} = 4a$ $\angle CAB = \sin^{-1} 2a\sqrt{3}/4a$ or $\cos^{-1} 2a/4a$ or $\tan^{-1} 2a\sqrt{3}/2a$ $= 60^\circ$ so $\angle SAB = 30^\circ$	A.G.	M1 A1	
	(ii)	<i>EITHER</i> Resolve vertically and horizontally, e.g.: (F_A may be in either direction) Eliminate $N_B + F_A$ to find N_A : <i>OR</i>	$\frac{1}{2} N_A + \frac{1}{2}\sqrt{3} N_B + \frac{1}{2}\sqrt{3} F_A = W$ and $\frac{1}{2}\sqrt{3} N_A = \frac{1}{2} N_B + \frac{1}{2} F_A$ $N_A = \frac{1}{2} W$	A.G.	M1 A1 A1	4
	(iii)	Resolve in dirn. PQ to find N_A : Second resolution, e.g. in dirn. PS : Take moments, e.g. about A : (A1 for each side of eqn) Solve to find N_B : Use N_B to find F_A :	$N_A = \frac{1}{2} W$ $N_B + F_A = \frac{1}{2}\sqrt{3} W$ $\frac{1}{2}\sqrt{3} W \times 3a/2 + \frac{1}{2} W \times (2\sqrt{3} - 3)a$ $= N_B \times 2a$ $N_B = \{(7\sqrt{3} - 6)/8\} W$ $F_A = \sqrt{3} N_A - N_B$ or $\frac{1}{2}\sqrt{3} W - N_B$ $= \{3(2 - \sqrt{3})/8\} W$ (A.E.F.)	A.G.	(M1 A1) (A1) M1 A1 A1 M1 A1 M1 A1	3 7
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B	Estimate population variance:	$s_P^2 = (236.0 - 42 \cdot 8^2 / 8) / 7$			
	(allow biased here: 0.8775 or 0.9367 ²)	$= 351 / 350$ or 1.003 or 1.001 ²	M1		
	Find confidence interval (allow z in place of t) e.g.:	$42.8 / 8 \pm t \sqrt{s_P^2 / 8}$	M1		
	Use correct tabular t -value:	$t_{7, 0.975} = 2.365$	A1		
	Evaluate C.I. correct to 2 d.p.:	5.35 ± 0.84 or [4.51, 6.19]	A1	4	
	Formulate inequality for k (or equality for k_{\max}):	$(5.35 - k) / \sqrt{s_P^2 / 8} \geq [\text{or } >] t$	M1		
	Use correct tabular t -value:	$t_{7, 0.9} = 1.415$	A1		
	Solve for k_{\max} (A0 if = or \leq was used for k above):	$5.35 - k \geq 0.50, k_{\max} = 4.85$	A1	3	
	State hypotheses (B0 for \bar{x} ...), e.g.:	$H_0: \mu_P = \mu_Q, H_1: \mu_P > \mu_Q$	B1		
	State assumption (A.E.F.):	Normal distns. for [P and] Q <i>and</i> equal variances	B1		
Estimate (pooled) common variance:	$s^2 = (7 \times 1.003 + 11 \times 1.962) / 18$ $= 1.589$ or 1.261 ²	M1 A1			
Calculate value of t (to 3 s.f.):	$t = (5.35 - 4.60) / (s \sqrt{1/8 + 1/12})$ $= 1.30$	M1 A1			
Correct conclusion (A.E.F., $\sqrt{\dagger}$ on t):	$t < t_{18, 0.9} = 1.33$ so Q 's mean is not less than P 's	B1 $\sqrt{\dagger}$	7	14	