



Cambridge International AS & A Level

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MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

October/November 2022

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- 5 (a) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z + 2| \leq 2$ and $\text{Im } z \geq 1$. [4]

- (b) Find the greatest value of $\arg z$ for points in the shaded region. [2]

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(b) Hence solve the equation $\sqrt{5} \sec 2x + \tan 2x = 4$, for $0^\circ < x < 180^\circ$. [4]

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8 The curve with equation $y = \frac{x^3}{e^x - 1}$ has a stationary point at $x = p$, where $p > 0$.

(a) Show that $p = 3(1 - e^{-p})$. [3]

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(b) Verify by calculation that p lies between 2.5 and 3. [2]

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(c) Use an iterative formula based on the equation in part (a) to determine p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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9 With respect to the origin O , the position vectors of the points A , B and C are given by

$$\vec{OA} = \begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix}, \quad \vec{OB} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \vec{OC} = \begin{pmatrix} 4 \\ -3 \\ -2 \end{pmatrix}.$$

The midpoint of AC is M and the point N lies on BC , between B and C , and is such that $BN = 2NC$.

(a) Find the position vectors of M and N . [3]

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(b) Find a vector equation for the line through M and N . [2]

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- 10** A gardener is filling an ornamental pool with water, using a hose that delivers 30 litres of water per minute. Initially the pool is empty. At time t minutes after filling begins the volume of water in the pool is V litres. The pool has a small leak and loses water at a rate of $0.01V$ litres per minute.

The differential equation satisfied by V and t is of the form $\frac{dV}{dt} = a - bV$.

- (a) Write down the values of the constants a and b . [1]

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- (b) Solve the differential equation and find the value of t when $V = 1000$. [6]

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(c) Obtain an expression for V in terms of t and hence state what happens to V as t becomes large. [2]

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11 Let $f(x) = \frac{5 - x + 6x^2}{(3 - x)(1 + 3x^2)}$.

(a) Express $f(x)$ in partial fractions.

[5]

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