Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

1472641323

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

JC21 11_9709_33/RP

[Turn over

1 Find the quotient and remainder when $2x^4 + 1$ is divided by $x^2 - x + 2$. [3]

2 **(a)** Sketch the graph of $y = |2x - 3|$. [1]

(b) Solve the inequality $|2x - 3| < 3x + 2$. [3]

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3 Solve the equation $4^{x-2} = 4^x - 4^2$, giving your answer correct to 3 decimal places. [4]

4 Find the exact value of $\int_{\frac{1}{3}\pi}^{\pi} x \sin \frac{1}{2}x \, dx$. [5]

5 Solve the equation $\sin\theta = 3\cos 2\theta + 2$, for $0° \leqslant \theta \leqslant 360°$. [5]

6 **(a)** By first expanding $\cos(x - 60°)$, show that the expression

$$2\cos(x - 60°) + \cos x$$

can be written in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 2 decimal places. [5]

(b) Hence find the value of x in the interval $0° < x < 360°$ for which $2\cos(x - 60°) + \cos x$ takes its least possible value. [2]

7 The equation of a curve is $\ln(x + y) = x - 2y$.

(a) Show that $\frac{dy}{dx} = \frac{x + y - 1}{2(x + y) + 1}$. [4]

(b) Find the coordinates of the point on the curve where the tangent is parallel to the x-axis. [3]

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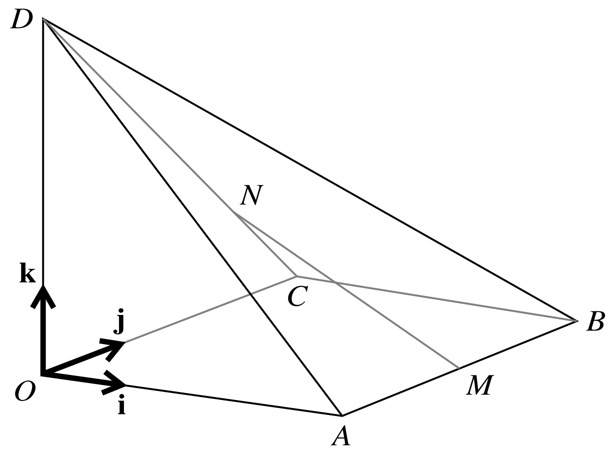

In the diagram, $OABCD$ is a pyramid with vertex D. The horizontal base $OABC$ is a square of side 4 units. The edge OD is vertical and $OD = 4$ units. The unit vectors **i**, **j** and **k** are parallel to OA, OC and OD respectively.

The midpoint of AB is M and the point N on CD is such that $DN = 3NC$.

(a) Find a vector equation for the line through M and N. [5]

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(b) Show that the length of the perpendicular from O to MN is $\frac{1}{3}\sqrt{82}$. [4]

9 Let $f(x) = \dfrac{1}{(9-x)\sqrt{x}}$.

(a) Find the x-coordinate of the stationary point of the curve with equation $y = f(x)$. [4]

(b) Using the substitution $u = \sqrt{x}$, show that $\int_0^4 f(x)\,dx = \frac{1}{3}\ln 5$. [6]

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10 A large plantation of area $20\,\text{km}^2$ is becoming infected with a plant disease. At time t years the area infected is $x\,\text{km}^2$ and the rate of increase of x is proportional to the ratio of the area infected to the area not yet infected.

When $t = 0$, $x = 1$ and $\dfrac{dx}{dt} = 1$.

(a) Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = \frac{19x}{20 - x}.$$ [2]

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(b) Solve the differential equation and show that when $t = 1$ the value of x satisfies the equation $x = e^{0.9+0.05x}$. [5]

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(c) Use an iterative formula based on the equation in part **(b)**, with an initial value of 2, to determine x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(d) Calculate the value of t at which the entire plantation becomes infected. [1]

11 The complex number $-\sqrt{3} + \mathrm{i}$ is denoted by u.

(a) Express u in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of r and θ. [2]

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(b) Hence show that u^6 is real and state its value. [2]

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(c) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $0 \leqslant \arg(z - u) \leqslant \frac{1}{4}\pi$ and $\operatorname{Re} z \leqslant 2$. [4]

(ii) Find the greatest value of $|z|$ for points in the shaded region. Give your answer correct to 3 significant figures. [2]

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Additional Page

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