Cambridge International AS & A Level

MATHEMATICS	**9709/42**
Paper 4 Mechanics	**October/November 2021**
MARK SCHEME	
Maximum Mark: 50	

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$\frac{20-6}{50-T}=\frac{20}{5}$ or $20=6+\frac{20}{5(50-T)}$	M1	Equate the accelerations and set up an equation in T. Allow correct use of *their* incorrect $\frac{20}{5}$.
	$T = 46.5$	A1	
		2	
1(b)	Distance = $\frac{1}{2}\times 5\times 20+20\times 20+\frac{1}{2}\times 5\times(20+6)+$ $+6\times(T-30)+\frac{1}{2}\times(50-T)\times(20+6)+\frac{1}{2}\times 10\times 20$ [= 50 + 400 + 65 + 99 + 45.5 + 100] OR Distance = $\frac{1}{2}\times 20\times(60+45)-\frac{1}{2}\times 14\times(25+T-30)$ [= 1050 – 290.5]	M1	Attempt to find the total distance travelled using areas. Allow with T not yet substituted. Allow one error in use of area formulae or omission of only one of the areas: 0–5, 5–25, 25–30, 30–T, T–50, 50–-60.
	Total distance travelled = 759.5 m	A1 FT	FT *their* T value: Provided $30 < T < 50$ and distance $=1085-7T$
		2	

Question	Answer	Marks	Guidance
2(a)	For van: $2500 - 700 - T = 3600a$ For trailer: $T - 300 = 1200a$ For system: $2500 - 700 - 300 = (3600 + 1200)a$	M1	Apply Newton's 2nd law to the van or to the trailer or to the system of van and trailer. Correct number of terms.
		A1	For any two correct.
	Obtain an equation in T only $\left[a = \frac{5}{16} = 0.3125\right]$	M1	
	Tension in the rope = $T = 675$ N	A1	
		4	
2(b)	For van: $-F - 700 = 3600a$ For trailer: $-300 = 1200a$ System: $-F - 700 - 300 = (3600 + 1200)a$	M1	Apply Newton's 2nd law to any two of the van, the trailer and the system with braking force F and with $T = 0$.
	Least possible value of braking force = $F = 200$ N	A1	Allow $F = -200$
		2	

Question	Answer	Marks	Guidance
3(a)	$mg \times 1.8 = \frac{1}{2}mv^2$	M1	Use of conservation of energy, 2 terms. Must NOT use constant acceleration equations. Use of equations such as $v^2 = u^2 + 2as$ scores **M0 A0.**
	Speed of block at $B = v = 6 \text{ ms}^{-1}$	A1	**AG**
		2	
3(b)	Attempt the work-energy equation	M1	In the form: ± KE lost = ± PE gain ± WD against Resistance
	$\frac{1}{2} \times m \times 6^2 = 4.5 + mg \times 1.2$	A1	If using motion from A to final point $mg \times 1.8 = mg \times 1.2 + 4.5$
	Mass of the block $= m = 0.75$ kg	A1	
		3	

Question	Answer	Marks	Guidance
4(a)	For differentiation of s	*M1	
	$v = 0.004(150t - 3t^2)$ [$= 0.6t - 0.012t^2$]	A1	
	$v = 0$ when $t = 50$. At $t = 50$, $s = 0.004(75 \times 50^2 - 50^3) = 0.3 \times 50^2 - 0.004 \times 50^3$	DM1	Solve $v = 0$ for t and substitute this value into s.
	Distance $AB = 250$ m	A1	**AG**
		4	

Question	Answer	Marks	Guidance
4(b)	Attempt to determine stationary points for v by differentiation or by use of symmetry [$a = 0.004(150 - 6t) = 0.6 - 0.024t$] or using symmetry attempt to find the mid-point between $t = 0$ and *their* t value at $v = 0$	*M1	If symmetry used then an attempt to find the required mid-point must be seen.
	Maximum v when $a = 0$ so $t = 25$ Or finding the mid-point if symmetry is used e.g. $v = 0.004(150 \times 25 - 3 \times 25^2) = 0.6 \times 25 - 0.012 \times 25^2$ [$= 7.5\ \text{ms}^{-1}$]	DM1	Attempt to solve $a = 0$ or use symmetry to find the relevant t value.
	Maximum velocity = $7.5\ \text{ms}^{-1}$	A1	
	Alternative method for question 4(b)		
	Attempt to velocity as $v = -0.012\left[(t-25)^2 - 25^2\right]$	M1*	Attempt to complete the square for *their* velocity as far as $k\left[(t-a)^2 - a^2\right]$
	$v = -0.012(t-25)^2 + 0.012 \times 25^2$ and select $t = 25$ as the maximum point.	DM1	Or select the 0.012×25^2 term as the maximum velocity.
	Maximum $= [0.012 \times 625 =] 7.5\ \text{ms}^{-1}$	A1	
		3	

Question	Answer	Marks	Guidance
5(a)	Driving force = $DF = \frac{960\,000}{30}$	B1	Allow for 960 000 = DF × 30
	$DF - 75000g \times \sin\alpha - R = 0$	M1	Resolve forces along the slope. Must use a value for either $\sin\alpha$ or α.
	Resistance force = R = 24 500 N	A1	Allow correct work with 24500 to 3 sf.
		3	
5(b)	WD by engine in 60 s = 900 000 × 60 [= 54000000]	B1	
	$KE_{init} = \frac{1}{2} \times 75000 \times 30^2 \quad KE_{final} = \frac{1}{2} \times 75000 \times v^2$	B1	For either correct expression for KE.
	$900\,000 \times 60 + \frac{1}{2} \times 75000 \times 30^2 = 46\,500\,000 + \frac{1}{2} \times 75000 \times v^2$	M1	For use of the work-energy equation with 4 terms, correct dimensions.
	Speed of engine after 60 s = v = 33.2 ms^{-1}	A1	Allow $v = \sqrt{1100} = 10\sqrt{11}$
		4	

Question	Answer	Marks	Guidance
6(a)	Horizontal: $100 - T_U \sin 60 - T_L \sin 60 = 0$ Vertical: $T_U \cos 60 - T_L \cos 60 - 5g = 0$ Perp to T_U $T_L \cos 30 + 5g \cos 30 = 100 \cos 60$	M1	Resolve horizontally or vertically or perpendicular to the upper string to reach an equation. Correct number of terms, Allow X for 100 in horizontal equation.
		A1	Either horizontal and vertical equations correct or perpendicular correct. Must see $X = 100$ used for A1.
	Solve for either T_L or T_U using equation(s) with no missing term.	M1	May see $T_U = 107.74$
	$T_L = 7.74$ N	A1	Allow 7.73
		4	
6(b)	Horizontal: $X - T_{up} \sin 60 = 0$ Vertical: $T_{up} \cos 60 - 5g = 0$ Perp to T_{up} $5g \cos 30 = X \cos 60$	M1	Resolve either horizontally or vertically or perpendicular to the upper string. Must be using the tension $T_{low} = 0$. Equivalent to Lami as: $\frac{5g}{\sin 150} = \frac{X}{\sin 120} \left(= \frac{T_{up}}{\sin 90} \right)$
		A1	Either horizontal and vertical equations correct or perpendicular correct.
	Eliminate T_{up} and/or solve for X	M1	$T_{up} = 100$
	Least value of $X = 86.6$	A1	Allow $X = 50\sqrt{3}$
		4	

Question	Answer	Marks	Guidance
7(a)	For Q: $-2mg \sin \alpha - F = 2ma$ [$-16m - 7.2m = 2ma$] $R = 2mg \cos \alpha$ [$= 12m$]	M1	Apply Newton's 2nd law along or perpendicular to the plane to particle Q. Must use values for α or sin α or cos α.
		A1	Both correct.
	$F = 0.6 \times 2mg \cos \alpha = 0.6 \times 0.6 \times 20m$ [$= 7.2m$] [$2(m)a = -2(m)g\ (0.8) - 0.6 \times 2(m)g\ (0.6)$]	M1	Using $F = 0.6R$ where R is a component of $2mg$ only
	Acceleration of Q up the plane while moving up the plane is $a = -11.6 \text{ ms}^{-2}$	A1	**AG**
		4	
7(b)	For P: $mg \sin \alpha - 0.6R = ma$, leading to $8m - 3.6m = ma$ [$R = mg\cos\alpha = 6m$, $a = 4.4 \text{ ms}^{-2}$]	M1	Apply Newton's 2nd law to attempt to find the acceleration of particle P. Must use values for α or sin α.
	Q comes to rest when $10 - 11.6T_1 = 0$, $\left[T_1 = \frac{25}{29} = 0.862\right]$	M1	For using constant acceleration equations to attempt to determine when $v_Q = 0$.
	For P $s_{P(\text{down})} = \frac{1}{2} \times 4.4 \times T_1^2$ $[= 1.635]$ For Q $s_{Q(\text{up})} = 10T_1 + \frac{1}{2} \times (-11.6) \times T_1^2$ $[= 4.31]$	M1	Use constant acceleration equations to attempt to find either $s_{P(\text{down})}$ or $s_{Q(\text{up})}$ at time T_1.
	$d = 6.4 - s_{P(down)} - s_{Q(up)}$ $[= 0.455]$ and to find T_2 $[= 0.12]$ by using $d = s_{P2} - s_{Q2} = (4.4T_1) \times T_2$ [s_{P2} and s_{Q2} are distances travelled by P and Q in time T_2]	M1	For attempting to find the extra distance $d[= 0.455]$ needed to reach 6.4 m and using $u_P = 4.4T_1$ at T_1 to find T_2 as $d = (4.4T_1)T_2 + \frac{1}{2} \times 4.4T_2^2 - \frac{1}{2} \times 4.4T_2^2$.
	Time before collision = $[t = T_1 + T_2 = 0.862 + 0.12 =]0.982$	A1	$t = 0.98194357\ldots$

Question	Answer	Marks	Guidance
7(b)	**Alternative method for Question 7(b)**		
	For P: $mg\sin\alpha - 0.6R = ma$, leading to $8m - 3.6m = ma$ $[R = mg\cos\alpha = 6m, a = 4.4\text{ ms}^{-2}]$	**M1**	Apply Newton's 2nd law to attempt to find the acceleration of particle P. Must use values for α or $\sin\alpha$
	Q comes to rest when $10 - 11.6T_1 = 0$, $\left[T_1 = \frac{25}{29} = 0.862\right]$	**M1**	For using constant acceleration equations to attempt to determine when $v_Q = 0$
	For P $s_{P(\text{down})} = \frac{1}{2}\times 4.4\times t^2$ For Q $s_{Q(\text{up})} = 10T_1 + \frac{1}{2}\times(-11.6)T_1^2 - \frac{1}{2}\times 4.4(t - T_1)^2$	**M1**	Use constant acceleration equations to attempt to find either $s_{P(\text{down})}$ or $s_{Q(\text{up})}$ at time t where t is the total time before collision.
	$\frac{1}{2}\times 4.4t^2 + 10T_1 + \frac{1}{2}\times(-11.6)T_1^2 - \frac{1}{2}\times 4.4(t - T_1)^2 = 6.4$	**M1**	For using $s_{P(\text{down})} + s_{Q(\text{up})} = 6.4$ and solving for t
	Time before collision is $t = 0.982$ s	**A1**	$t = 0.98194357\ldots$
		5	
	Special case for those who do not take into account the fact that Q comes to rest and then changes its direction		
	For P: $mg\sin\alpha - 0.6R = ma$, leading to $8m - 3.6m = ma$ $[R = mg\cos\alpha = 6m, a = 4.4\text{ ms}^{-2}]$	**M1**	Apply Newton's 2nd law to attempt to find the acceleration of particle P. Must use values for α or $\sin\alpha$.
	For P $s_{p(\text{down})} = (\pm)\frac{1}{2}\times 4.4t^2$ For Q $s_{q(\text{up})} = (\pm)\ 10t + \frac{1}{2}\times(-11.6)t^2$	**M1**	For using constant acceleration equations to attempt to find either $s_{p(\text{down})}$ or $s_{q(\text{up})}$.
	$s_p + s_q = 6.4$ leading to $\frac{1}{2}\times 4.4t^2 + 10t + \frac{1}{2}\times(-11.6)t^2 = 6.4$	**M1**	For applying $(\pm)\ s_p + (\pm)\ s_q = 6.4$ using their expressions for s_p and s_q to set up and solve a 3-term quadratic equation in t to obtain at least 1 solution.

Question	Answer	Marks	Guidance
7(b)	Time that particles are in motion before collision = $t = 1$ s	A1	Must reject $t = 16/9$ **Maximum mark 4 out of 5**
		4	
7(c)	$u_{p(\text{down})} = 0 + 4.4 \times 0.982$ [= 4.3208]	B1 FT	Allow ±4.4. FT on *their* 4.4 and *their* 0.982
	$u_{q(\text{down})} = 4.4 \times 0.12$ [= 0.528]	B1 FT	Allow ±4.4. FT on *their* 4.4 and *their* 0.12
	$\pm m \times 4.3208 \pm 2m \times 0.528 = \pm(m + 2m)v$ [Correct equation is $m \times 4.3208 + 2m \times 0.528 = \pm(m + 2m)v$]	M1	Apply conservation of momentum, 4 terms, using *their* u_p and u_q values with m and $2m$ respectively. Velocity of P and Q after impact must be equal.
	Speed of combined particle immediately after impact = $v = 1.79\ \text{ms}^{-1}$	A1	Must be positive
	Special case for those who do not take into account the fact that *Q* comes to rest and then changes its direction		
	$u_{p(\text{down})} = 0 + 4.4 \times 1$ [= 4.4]	B1 FT	Allow ±4.4, FT on *their* 1 and *their* 4.4
	$u_{q(\text{up})} = 10 - 11.6 \times 1$ [= –1.6] so $u_{q(\text{down})} = 1.6$	B1 FT	Allow ± (10 – 11.6 × 1), FT on *their* 1
	$\pm m \times 4.4 \pm 2m \times 1.6 = \pm(m + 2m)v$	M1	Apply conservation of momentum, 4 terms, using their u_p and u_q values with m and $2m$ respectively. Velocity of P and Q after impact must be equal.
	Speed of combined particle immediately after impact = $v = 2.53\ \text{ms}^{-1}$	A1	Allow $v = \frac{38}{15}$. Must be positive.
		4	