UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2011 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/13 Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

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International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 $k^3 \times \left(\frac{1}{3}x\right)^2 \times 10$ (or correct factorials) $10 \times k^3 \times \frac{1}{9} = 30 \Rightarrow k = 3$	B2 B1 [3]	B1 for 2/3 terms correct cao
2 **(i)** 5[8 + 9 × 4] 220 **(ii)** $\dfrac{4\left(2^{10}-1\right)}{2-1}$ 4092	M1 A1 [2] M1 A1 [2]	Use correct formula with a=4, d=4 Use correct formula with a=4, r=2 or ½ 4090 without 4092 A0
3 **(i)** $2x^5 + 3x^2 = 2x \Rightarrow 2x^5 + 3x^2 - 2x = 0$ $[x(2x]^4 + 3x^2 - 2) = 0$ $2x^4 + 3x^2 - 2 = 0$ **(ii)** $(x^2 + 2)(2x^2 - 1) = 0$ $x = \pm\ 1/\sqrt{2}$ only $\left(1/\sqrt{2},\ 2/\sqrt{2}\right), \left(-1/\sqrt{2},\ -2/\sqrt{2}\right)$	M1 A1 [2] M1 A1 A1 [3]	First line essential **AG** Factorising needed for A1 Reasonable attempt at solving a quadratic in x^2 For a correct pair of solutions, either 2 x's or 1 x and 1 y SC (±0.707, ±1.41) AWRT B1
4 **(i)** $10^2 \sin 0.8 = 71.7$ **(ii)** sector(s) = (2) × $\frac{1}{2}$ × 10^2 × 0.8 = (2) × 40 Total area = 80 **(iii)** arc(s) = (2) × 10 × 0.8 16+20 = 36	M1A1 [2] M1 A1 [2] M1 A1 [2]	Completely correct method for a triangle Correct formula used for a sector Correct formula used for an arc
5 **(i)** $3\cos^2 x + 8\cos x + 4 = 0$ $(3\cos x + 2)(\cos x + 2) = 0$ $\cos x = -\dfrac{2}{3}$ **(ii)** $\cos(\theta + 70) = -\dfrac{2}{3}$, $\theta = 61.8$ $\theta + 70 = 131.8$ (or 228.2) $\theta = 158.2$	M1 M1 A1 [3] M1 A1 M1 A1 [4]	Use of $c^2 + s^2 = 1$ Factorising, formula or completing the square needed **AG** Ignore $\cos x = -2$ also offered SC B1 if –2/3 and –2 seen

6 **(i)** Scalar product = 15−8+3 10 = |**OA**| |**OB**| cos θ |**OA**| = √26, |**OB**| = √38 Angle *BOA* = 71.4 or 71.5 or 1.25 radians	M1 M1 M1 A1 [4]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ Correct magnitude for either Linking everything correctly cao
(ii) **a**+½(**b**–**a**) or **b**+½(**a**–**b**) or ½(**a**+**b**) –2**b** + *their* **c** oe –6**i** + 5**j** + 4**k**	M1 M1 A2,1,0 [4]	
7 **(i)** y = m(x – 2) oe	B1 [1]	Accept y = mx + c, c = – 2m
(ii) $x^2 - 4x + 5 = mx - 2m \Rightarrow x^2 - x(4 + m) + 5 + 2m = 0$ $(4 + m)^2 - 4(5 + 2m) = 0 \Rightarrow m^2 - 4 = 0$ m = ±2 $m = 2 \Rightarrow x^2 - 6x + 9 = 0 \Rightarrow x = 3$ $m = -2 \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow x = 1$ (3, 2), (1, 2)	M1 DM1 A1 DM1 A1 A1 [6]	Apply $b^2 - 4ac$ Substitute their m and attempt to solve for x Allow for a pair of x values or 1 x and 1 y.
OR m = 2x – 4 **y = mx – 2m, y = x^2 – 4x + 5**	M1 M1 M1 A1 A1 A1	**Eliminating 2 variables from 3 equations.** **Obtaining a quadratic in x or y.** **Solving their quadratic correctly.** A pair of x values or 1 x and 1 y.. m=2,–2 also needed for final mark.
(iii) $(x - 2)^2 + 1$, (2, 1)	B1,B1 [2]	

8 **(i)**	$f'(3) = 0 \Rightarrow 18 + 3k - 12 = 0$ $k = -2$ $(x-3)(x+2) = 0$ $x = -2$, (Allow also = 3)	M1 A1 M1 A1 [4]	 AG
(ii)	$f''(x) = 4x - 2$ $f''(3) > 0$ hence min at P $f''(-2) < 0$ hence max at Q	B1 B1 [2]	 3 min, –2 max independent of $f''(x)$
(iii)	$f(x) = \frac{2}{3}x^3 - x^2 - 12x\ (+\ c)$ Sub $(3, -10) \rightarrow -10 = 18 - 9 - 36 + c$ $c = 17$	B2,1,0 M1 A1 [4]	Accept anywhere in question Dependent on c present Condone y =, or equation =
9 **(i)**	$f^{-1}(x) = \frac{1}{2}x - \frac{3}{2}$ $2x + 3 = \frac{1}{2}x - \frac{3}{2} \Rightarrow x = -3$	B1 M1A1 [3]	
(ii)	2 lines approximately correct, reflected in $y=x$ & meeting at $(-3, -3)$	B3,2,1,0 [3]	Can be implied by graph or in writing. Ignore lines extended
(iii)	$gf(x) = (2x+3)^2 - 6(2x+3)$ $4x^2 - 9$ $4x^2 - 9 \le 16 \Rightarrow x^2 \le \frac{25}{4}$ $-\frac{5}{2} \le x \le 0$	M1 A1 M1 A1A1 [5]	 Solving any quadratic to do with f and g ≤16, to x = Condone < and >

10 **(i)** $\int (x+1)^{\frac{1}{2}} - (x+1)$ or $\int (y^2-1)-(y-1)$	M1	Dealing with line as a triangle or integral with correct limits.
$\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{1}{2}x^2 - x$ or $\frac{1}{3}y^2 - \frac{1}{2}y^2$	M1A1	Attempt at integral of curve.
$\frac{2}{3} - \left(0 - \frac{1}{2} + 1\right)$ or $\frac{1}{3} - \frac{1}{2}$	DM1	Applying limits $-1 \rightarrow 0$ or $0 \rightarrow 1$ to curve
$\frac{1}{6}$	A1 [5]	π included loses last mark.
(ii) $V_1 = (\pi)\int (y^2-1)^2 = (\pi)\int y^4 - 2y^2 + 1$	M1	Attempt at $\int x^2$ dy for curve
$(\pi)\left[\frac{y^5}{5} - \frac{2y^2}{3} + y\right]$	A1	
$(\pi)\left[\frac{1}{5} - \frac{2}{3} + 1\right]$	DM1	Apply limits $0 \rightarrow 1$
$V_1 = \frac{8}{15(\pi)}$ or 0.533(π) (AWRT)	A1	
or $(\pi)\left[\frac{y^3}{3} - y^2 + y\right]$	M1	Or $\frac{1}{3} \times \pi(\times 1^2 \times 1)$
$V_2 = \frac{1}{3}\pi$	A1	Vol of cone or attempt to $\int x^2 dy$ for line
Volume = $\frac{8}{15}\pi \quad \frac{1}{-3}\pi = \frac{1}{5}\pi$ (or 0.628)	A1 [7]	
OR $(y^4 - 2y^2 + 1) - (y^2 - 2y + 1)$	M1	Attempt to $\int x^2 dy$
$(\pi)\int y^4 - 3y^2 + 2y$	M1	Attempt to $\int (x_1^2 - x_2^2)$
$(\pi)\left[y^{\uparrow}5/5 - y^{\uparrow}3 + y^{\uparrow}2\right]$	A1,A1,A1	
$(\pi)\left[\frac{1}{5} - 1 + 1\right]$	DM1	Apply limits $0 \rightarrow 1$ dependent on first M1
$\frac{1}{5}\pi$	A1	

$\int_{-1}^{0} x+1-\int_{-1}^{0}\left(x+1)\right)^2$	M1	SC MR integrating about x axis
$\left[\frac{x^2}{2}+x\right]-\left[\frac{x+1^3}{3}\right]$		
SC $=\left[(0)-\left(\frac{1}{2}-1\right)\right]-\left[\frac{1}{3}-0\right]$	M1	Use of –1,0 as limits
$\frac{1}{2}-\frac{1}{3}=\frac{1}{6}\pi$ (0.524)	A1	