



## Cambridge International AS & A Level

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**MATHEMATICS**

**9709/31**

Paper 3 Pure Mathematics 3

**May/June 2021**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

1 Solve the inequality  $2|3x - 1| < |x + 1|$ .

[4]

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- 2 Find the real root of the equation  $\frac{2e^x + e^{-x}}{2 + e^x} = 3$ , giving your answer correct to 3 decimal places. Your working should show clearly that the equation has only one real root. [5]

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- 3 (a) Given that  $\cos(x - 30^\circ) = 2 \sin(x + 30^\circ)$ , show that  $\tan x = \frac{2 - \sqrt{3}}{1 - 2\sqrt{3}}$ . [4]

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- (b) Hence solve the equation

$$\cos(x - 30^\circ) = 2 \sin(x + 30^\circ),$$

for  $0^\circ < x < 360^\circ$ .

[2]

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- 4 (a) Prove that  $\frac{1 - \cos 2\theta}{1 + \cos 2\theta} \equiv \tan^2 \theta$ . [2]

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- (b) Hence find the exact value of  $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \frac{1 - \cos 2\theta}{1 + \cos 2\theta} d\theta$ . [4]

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5 (a) Solve the equation  $z^2 - 2piz - q = 0$ , where  $p$  and  $q$  are real constants. [2]

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In an Argand diagram with origin  $O$ , the roots of this equation are represented by the distinct points  $A$  and  $B$ .

(b) Given that  $A$  and  $B$  lie on the imaginary axis, find a relation between  $p$  and  $q$ . [2]

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6 The parametric equations of a curve are

$$x = \ln(2 + 3t), \quad y = \frac{t}{2 + 3t}.$$

(a) Show that the gradient of the curve is always positive. [5]

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(b) Find the equation of the tangent to the curve at the point where it intersects the  $y$ -axis. [3]

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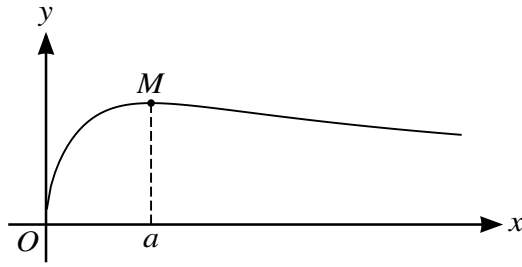
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The diagram shows the curve  $y = \frac{\tan^{-1} x}{\sqrt{x}}$  and its maximum point  $M$  where  $x = a$ .

(a) Show that  $a$  satisfies the equation

$$a = \tan\left(\frac{2a}{1+a^2}\right). \quad [4]$$

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(b) Verify by calculation that  $a$  lies between 1.3 and 1.5. [2]

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(c) Use an iterative formula based on the equation in part (a) to determine  $a$  correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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8 With respect to the origin  $O$ , the points  $A$  and  $B$  have position vectors given by  $\vec{OA} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$  and  $\vec{OB} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$ . The line  $l$  has equation  $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ .

(a) Find the acute angle between the directions of  $AB$  and  $l$ . [4]

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(b) Find the position vector of the point  $P$  on  $l$  such that  $AP = BP$ .

[5]

Dotted lines for writing the answer.

9 The equation of a curve is  $y = x^{-\frac{2}{3}} \ln x$  for  $x > 0$ . The curve has one stationary point.

(a) Find the exact coordinates of the stationary point. [5]

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(b) Show that  $\int_1^8 y \, dx = 18 \ln 2 - 9$ .

[5]

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10 The variables  $x$  and  $t$  satisfy the differential equation  $\frac{dx}{dt} = x^2(1 + 2x)$ , and  $x = 1$  when  $t = 0$ .

Using partial fractions, solve the differential equation, obtaining an expression for  $t$  in terms of  $x$ . [11]

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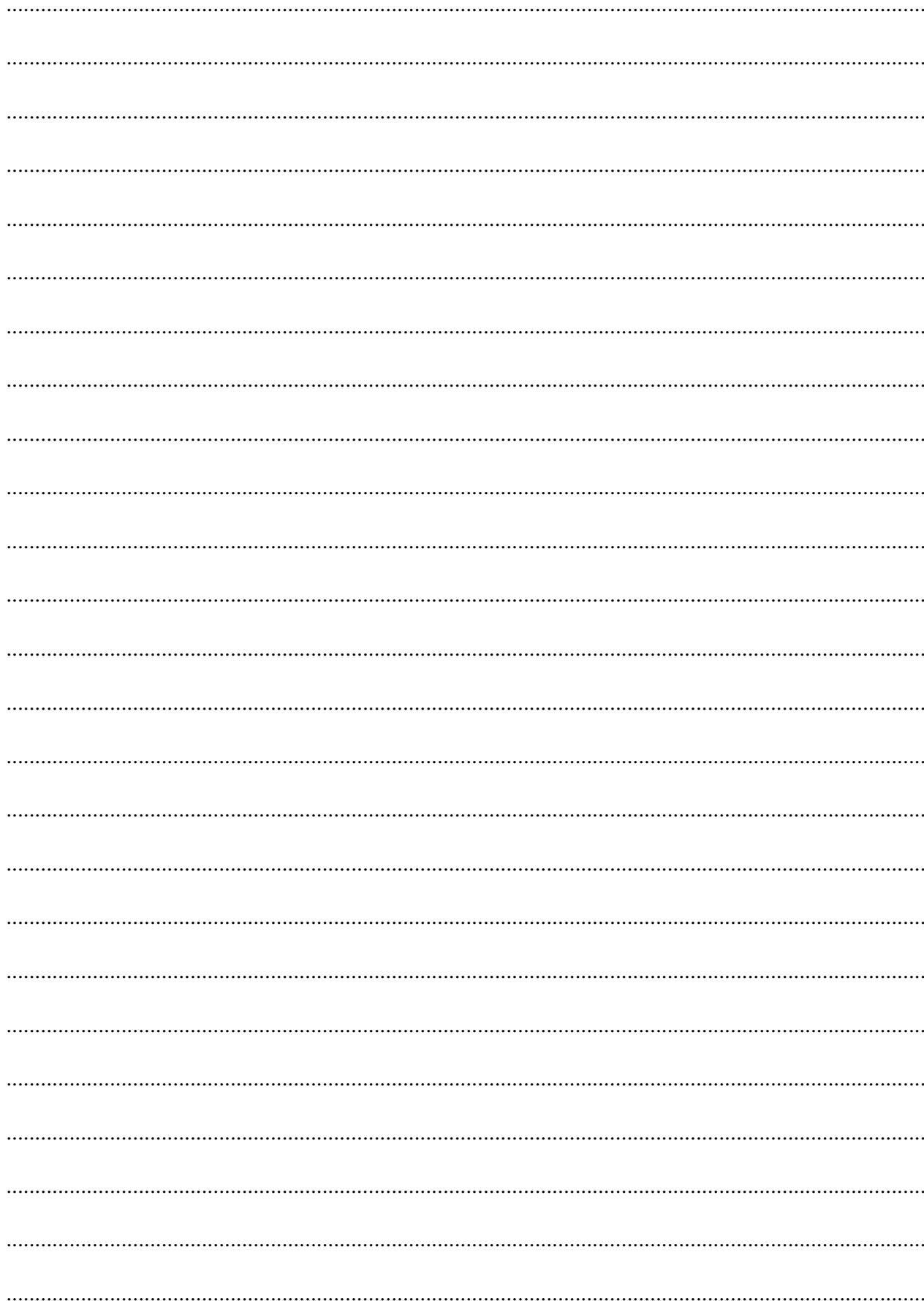
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**Additional Page**

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